# DYNAMIC ANALYSIS FOR END-OF-LIFE VEHICLES MANAGEMENT SYSTEMS: AN INEXACT MULTI-STAGE PROGRAMMING APPROACH

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***Abstract:*** *Nowadays, export of used vehicles represents the most significant barrier to the more efficient vehicle recycling in the EU, because millions of vehicles which are expected to go to domestic vehicle recycling factories are exported. As a result, how to allocate limited and frequently insufficient quantities of collected end-of-life vehicles (ELVs) to satisfy vast demands of vehicle recycling factories becomes a significant concern of many waste management systems that control the ELV collection and treatment networks across the EU. In this paper, a multi-stage interval-stochastic programming (MSISP) model is developed for supporting the management of ELV allocation under uncertainty. The MSISP is a hybrid of inexact optimization and multi-stage stochastic programming. The formulated MSISP model can directly handle uncertainties expressed as either probability density functions or discrete intervals. In addition, it can handle uncertainties through constructing a set of scenarios and reflect dynamic features of the system conditions.*

***Keywords****: End-of-life vehicle; Decision making; Multi-stage stochastic programming; Interval programming.*

## 1. INTRODUCTION

The amount of material passing through the end-of-life (ELV) recycling networks all over the EU has been reduced due to an increased export of used vehicles for reuse as second-hand vehicles or as sources of used parts and materials. Nowadays, export of used vehicles represents the most significant barrier to the more efficient vehicle recycling in the EU, because millions of vehicles, which are supposed to go to domestic vehicle recycling factories, are exported. Illegal treatment facilities represent another problem for the EU vehicle recycling industry sector, especially in some new EU member states (Tavoularis et al., 2009). Illegal operators can pay more to the last owners for an ELV, because they have no intention of incurring the cost of either depolluting the vehicle or attempting to reach the eco-efficiency targets promulgated by the EU ELV Directive (EU, 2000). Abandoned vehicles represent another major problem and the introduction of the EU ELV Directive initially increased their number. Consequently, how to allocate limited and often insufficient quantities of collected, decontaminated and flattened ELVs to satisfy vast demands of vehicle recycling factories becomes a significant concern of many waste management systems that control the ELV collection and treatment networks all over the EU.

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Owing to the complexity of the vehicle recycling subject, a very small number of research articles have been published. Reuter at al. (2006) formulated a nonlinear model to optimise the performances of the ELV recycling system. Qi and Hongcheng (2008) proposed a mixed integer linear programming model for designing ELV recovery network. Cruz-Rivera and Ertel (2009) constructed an uncapacitated facility location model in order to design a collection network for ELVs in Mexico. Vidovic et al. (2011) presented modelling approach that could be used to locate collection facilities for ELVs. Stoyanov (2012) formulated a multi-source capacitated facility location model in order to design a network of dismantling centers for ELVs in Bulgaria. Gołębiewski et al. (2013) proposed a simulation approach that could be used to determine locations for ELV dismantlers. Simic and Dimitrijevic (2013) developed a risk explicit interval linear programming model for optimal long-term planning in the EU vehicle recycling factories. Mora et al. (2014) proposed a mixed integer linear programming model for ELV closed-loop network design.

From the review of previous literature, it is evident that a number of systems analysis methods were developed for solving various ELV management problems. However, the above methods can hardly solve problems where multi-stage decisions need to be made, particularly when random variables exist in the vehicle recycling system while decisions have to be made before the random event occurs. In fact, no previous study was reported on the application of the multi-stage stochastic programming technique to ELV management problems. Therefore, in view of the limitations in previous works, the multi-stage interval-stochastic programming (MSISP) model is formulated and presented in this paper.

The remaining part of the paper is organized as follows: Section 2 describes the considered problem and presents the multi-stage interval-stochastic programming model for supporting the management of ELV allocation under uncertainty. Section 3 presents conclusions of the work.

## 2. METHODOLOGY

Consider a waste management system where a recycling manager is responsible for allocating ELVs from multiple regions to multiple vehicle recycling factories within a multi-period planning horizon. The available quantities of ELVs in each region are random variables with known probabilities. Decisions about ELV allocation targets must be made at an earlier stage when varying probability levels exist, because managers of all vehicle recycling factories need to know in advance which quantity of ELVs they can expect in order to create adequate production plans. Therefore, guaranteed quantities of ELVs from each region to each vehicle recycling factory are promulgated in advance. On the other hand, if the guaranteed quantities of ELVs cannot be distributed to vehicle recycling factories due to insufficient number of collected ELVs, vehicle recycling factories will have to import ELVs at a higher price and/or work at reduced capacity. To all vehicle recycling factories which have not fully been provided with preliminary ELV allocation targets penalties must be reimbursed, as specific compensation for acquiring ELVs from more expensive sources or because of smaller profit due to reduced working capacities.

The quantities of collected, depolluted and flattened ELVs during every planning period are random variables, and the appropriate ELV allocation plan would be of dynamic feature. As a result, ELV allocation decisions need to be made periodically. Thus, the problem of ELV allocation needs to be solved using a multi-stage stochastic programming approach, because it provides the possibility to represent various uncertainties in a form of multilayer scenario tree.

Uncertainties exist also in economic parameters, ELV allocation targets and demand from vehicle recycling factories. Observing aforementioned modelling parameters as interval values is purely natural. In order to reflect such uncertainties, interval-parameter programming needs to be introduced into the modelling formulation.

Thus, the described problem can be formulated as a multi-stage interval-stochastic programming model for planning end-of-life vehicles allocation as follows:

$$Max \quad f^{\pm} = \sum_{t=1}^{T} \sum_{r \in \mathrm{R}} \sum_{v \in \mathrm{V}} (D_{vrt}^{\pm}(Z_{vrt}^{-} + \Delta Z_{vrt}\gamma_{vrt}) - \sum_{s \in \mathrm{S}_t} p_{rst} K_{vt}^{\pm} M_{vrst}^{\pm}) \tag{1a}$$

subject to:

$$\sum_{v \in \mathrm{V}} (Z_{vrt}^{-} + \Delta Z_{vrt}\gamma_{vrt} - M_{vrst}^{\pm}) \le Q_{rst}^{\pm} + H_{rs't-1}^{\pm}, \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall s' \in \Gamma_s^{-1}; \forall t \in \{1,\ldots,T\} \tag{1b}$$

$$H_{r10}^{\pm} = \Pi_r^{\pm}, \ \forall r \in \mathrm{R} \tag{1c}$$

$$H_{rst}^{\pm} = Q_{rst}^{\pm} - \sum_{v \in \mathrm{V}} (Z_{vrt}^{-} + \Delta Z_{vrt}\gamma_{vrt} - M_{vrst}^{\pm}) + H_{rs't-1}^{\pm}, \quad \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall s' \in \Gamma_s^{-1}; \forall t \in \{1,\ldots,T\} \tag{1d}$$

$$\sum_{r \in \mathrm{R}} (Z_{vrt}^{-} + \Delta Z_{vrt}\gamma_{vrt} - M_{vrst}^{\pm}) \ge I_{vt\,min}^{\pm}, \ \forall v \in \mathrm{V}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{1e}$$

$$Z_{vrt}^{-} + \Delta Z_{vrt}\gamma_{vrt} \ge M_{vrst}^{\pm} \ge 0, \quad \forall v \in \mathrm{V}; \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{1f}$$

$$H_{rst}^{\pm} \ge 0, \ \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{0,1,\ldots,T\} \tag{1g}$$

$$0 \le \gamma_{vrt} \le 1, \ \forall v \in \mathrm{V}; \forall r \in \mathrm{R}; \forall t \in \{1,\ldots,T\} \tag{1h}$$

where: *t* is index of time period, $t \in \{1,\ldots,T\}$; R is set of considered regions; V is set of considered vehicle recycling factories; $\mathrm{S}_t$ is set of scenarios in period *t*; $\mathrm{S} = \bigcup_{t=1}^{T} \mathrm{S}_t$ is set of scenarios in planning horizon; $\mathrm{A} \subseteq \{(s',s) | s' \in \mathrm{S}, s \in \mathrm{S}\}$ is set of conse-cutive scenarios, which is determined in line with the structure of the scenario tree; $\Gamma_s^{-1} = \{s' | (s',s) \in \mathrm{A}\}, s' \in \mathrm{S}$ is set of scenarios that precede scenario *s*; *T* is number of analyzed time periods; $f^{\pm}$ is interval value of the expected profit to ELV management system over the planning horizon; $\Pi_r^{\pm}, \forall r \in \mathrm{R}$ is interval value of initial inventory weight of ELVs collected, depolluted, flattened and piled in region *r*; $I_{vt\,min}^{\pm}, v \in \mathrm{V}$ is interval value of safety inventory level of vehicle recycling factory *v* in period *t*; $D_{vrt}^{\pm}, v \in \mathrm{V}, r \in \mathrm{R}$ is interval value of revenue to ELV management system per weight unit of ELVs allocated from region *r* to vehicle recycling factory *v* in period *t*; $K_{vt}^{\pm}, v \in \mathrm{V}$ ($K_{vt}^{\pm} > D_{vrt}^{\pm}, \forall v,r,t$) is interval value of loss (i.e. penalty) to ELV management system per weight unit of ELVs not delivered to vehicle recycling factory *v* in period *t*; $p_{rst}, r \in \mathrm{R}, s \in \mathrm{S}_t$ is the probability of occurrence for scenario *s* in region *r* and period *t*; $Q_{rst}^{\pm}, r \in \mathrm{R}, s \in \mathrm{S}_t$ is interval value of quantity of ELVs collected, depolluted and flattened by authorized treatment facilities from region *r* in scenario *s* and period *t*; $Z_{vrt}^{\pm}, v \in \mathrm{V}, r \in \mathrm{R}$ is interval value of fixed ELV allocation target from region *r* to vehicle recycling factory *v* in period *t*; $\gamma_{vrt}, v \in \mathrm{V}, r \in \mathrm{R}$ is decision variable that is used for identifying an optimized set of ELV allocation targets $Z^{\pm}_{vrt} = Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt}$, where $\Delta Z_{vrt} = Z^{+}_{vrt} - Z^{-}_{vrt}$ and $\gamma_{vrt} \in [0,1]$; $M^{\pm}_{vrst}, v \in \mathrm{V}, r \in \mathrm{R}, s \in \mathrm{S}_t$ is interval value of quantity by which ELV allocation target prescribed between region *r* and vehicle recycling factory *v* is not met in scenario *s* and period *t*; $H^{\pm}_{rst}, r \in \mathrm{R}, s \in \mathrm{S}_t$ is interval value of weight of ELVs piled in region *r* and scenario *s* at the end of period *t*.

In Model (1), the objective function (1a) seeks to maximize the expected profit of the ELV management system through allocating ELVs from multiple regions to multiple vehicle recycling factories over a multi-stage context. In the objective function, the first term calculates revenue to ELV management system from allocating ELVs to vehicle recycling factories. The second term represents the loss (i.e. penalty) to ELV management system for violating the promulgated ELVs allocation targets. Constraints (1b) enforce that under all possible scenarios the quantity of ELVs allocated from some region to vehicle recycling factories handled by the ELV management system cannot be larger than the sum of quantity of piled ELVs across that region and the quantity of ELVs collected, depolluted and flattened by authorized treatment facilities located in that region. Constraints (1c) initialize inventories in regions which covers the considered ELV management system. Constraints (1d) enforce the inventory balances. Constraints (1e) ensure the safety inventory levels in vehicle recycling factories handled by the considered ELV management system in order to protect their shredders from starvation. Finally, constraints (2f)–(2h) define the value domains of decision variables used in the proposed model.

Model (1) can be decomposed into two deterministic sub-models corresponding to the lower and upper bounds of the desired objective value, and solved using an interactive algorithm. The sub-model corresponding to $f^{+}$ can be firstly formulated as follows:

$$Max \quad f^{+} = \sum_{t=1}^{T} \sum_{r \in \mathrm{R}} \sum_{v \in \mathrm{V}} (D^{+}_{vrt}(Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt}) - \sum_{s \in \mathrm{S}_t} p_{rst} K^{-}_{vt} M^{-}_{vrst}) \tag{2a}$$

subject to:

$$\sum_{v \in \mathrm{V}} \left( Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt} - M^{-}_{vrst} \right) \le Q^{+}_{rst} + H^{+}_{rs't-1}, \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall s' \in \Gamma^{-1}_{s}; \forall t \in \{1,\ldots,T\} \tag{2b}$$

$$H^{+}_{r10} = \Pi^{+}_{r}, \forall r \in \mathrm{R} \tag{2c}$$

$$H^{+}_{rst} = Q^{+}_{rst} - \sum_{v \in \mathrm{V}} (Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt} - M^{-}_{vrst}) + H^{+}_{rs't-1}, \quad \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall s' \in \Gamma^{-1}_{s}; \forall t \in \{1,\ldots,T\} \tag{2d}$$

$$\sum_{r \in \mathrm{R}} \left( Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt} - M^{-}_{vrst} \right) \ge I^{-}_{vt\,min}, \quad \forall v \in \mathrm{V}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{2e}$$

$$Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt} \ge M^{-}_{vrst} \ge 0, \quad \forall v \in \mathrm{V}; \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{2f}$$

$$H^{+}_{rst} \ge 0, \quad \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{0,1,\ldots,T\} \tag{2g}$$

$$0 \le \gamma_{vrt} \le 1, \quad \forall v \in \mathrm{V}; \forall r \in \mathrm{R}; \forall t \in \{1,\ldots,T\} \tag{2h}$$

where $M^{-}_{vrst}$, $H^{+}_{rst}$ and $\gamma_{vrt}$ are decision variables. Let $f^{+}_{opt}$, $M^{-}_{vrst\,opt}$, $H^{+}_{rst\,opt}$ and $\gamma_{vrt\,opt}$ be the solutions of sub-model (2). Then, the second sub-model corresponding to $f^{-}$ can be formulated as follows:

$$Max \quad f^{-} = \sum_{t=1}^{T} \sum_{r \in \mathrm{R}} \sum_{v \in \mathrm{V}} (D^{-}_{vrt}(Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt\,opt}) - \sum_{s \in \mathrm{S}_t} p_{rst} K^{+}_{vt} M^{+}_{vrst}) \tag{3a}$$

subject to:

$$\sum_{v \in \mathrm{V}} (Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt\,opt} - M^{+}_{vrst}) \leq Q^{-}_{rst} + H^{-}_{rs't-1},\ \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall s' \in \Gamma^{-1}_{s}; \forall t \in \{1,\ldots,T\} \tag{3b}$$

$$H^{-}_{r10} = \Pi^{-}_{r},\ \forall r \in \mathrm{R} \tag{3c}$$

$$\begin{aligned} &H^{-}_{rst} = Q^{-}_{rst} - \sum_{v \in \mathrm{V}} (Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt\,opt} - M^{+}_{vrst}) + H^{-}_{rs't-1}, \\ &\forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall s' \in \Gamma^{-1}_{s}; \forall t \in \{1,\ldots,T\} \end{aligned} \tag{3d}$$

$$\sum_{r \in \mathrm{R}} \left(Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt\,opt} - M^{+}_{vrst}\right) \geq I^{+}_{vt\,min},\ \forall v \in \mathrm{V}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{3e}$$

$$Z^{-}_{vrt} + \Delta Z_{vrt}\gamma_{vrt\,opt} \geq M^{+}_{vrst} \geq M^{-}_{vrst\,opt},\ \forall v \in \mathrm{V}; \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{3f}$$

$$0 \leq H^{-}_{rst} \leq H^{+}_{rst},\ \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{0,1,\ldots,T\} \tag{3g}$$

where $M^{+}_{vrst}$ and $H^{-}_{rst}$ are decision variables. Let $f^{-}_{opt}$, $M^{+}_{vrst\,opt}$ and $H^{-}_{rst\,opt}$ be solutions of sub-model (3). Thus, the primal solutions for Model (1) are:

$$f^{\pm}_{opt} = \left[f^{-}_{opt}, f^{+}_{opt}\right], \tag{4a}$$

$$M^{\pm}_{vrst\,opt} = \left[M^{-}_{vrst\,opt}, M^{+}_{vrst\,opt}\right],\ \forall v \in \mathrm{V}; \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{4b}$$

$$H^{\pm}_{rst\,opt} = \left[H^{-}_{rst\,opt}, H^{+}_{rst\,opt}\right],\ \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{4c}$$

The optimal ELV allocation scheme over the planning horizon is:

$$A^{\pm}_{vrst\,opt} = Z^{\pm}_{vrt} - M^{\pm}_{vrst\,opt},\ \forall v \in \mathrm{V}; \forall r \in \mathrm{R}; \forall s \in \mathrm{S}_t; \forall t \in \{1,\ldots,T\} \tag{4d}$$

where $A^{\pm}_{vrst\,opt}$ is calculated quantity of ELVs allocated between region *r* and vehicle recycling factory *v* in scenario *s* and period *t*.

## 3. CONCLUSIONS

This paper introduces the multi-stage interval-stochastic programming model for planning end-of-life vehicles allocation which is applicable across vehicle recycling industry that processes dozens of millions of ELVs every year. The formulated model is based on multi-stage stochastic programming and interval linear programming approaches. Thus, it can directly handle parameter uncertainties expressed as both probability density functions and discrete intervals.

The presented model is capable of incorporating multiple policies within the optimization framework. It permits comprehensive analyses of various policy situations that are associated with different levels of economic penalties and system failure risks when the promulgated ELV allocation targets are disregarded. Compared with the conventional multi-stage stochastic programming approach, the presented multi-stage interval-stochastic programming model for planning end-of-life vehicles allocation can incorporate much more uncertain information. Finally, it can be outlined that the proposed model is more than effective in tackling hard, uncertainty existing waste management problems. Future research will focus on extensive testing of the formulated model.

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