# BENEFITS FROM COOPERATION BEHAVIOUR IN VEHICLE ROUTING PROBLEM

**Zuzana Čičková [a] and Dana Figurová [b,*]**

[a] University of Economics in Bratislava, Faculty of Economic Informatics, Department of Operations Research and Econometrics, Slovakia

[b] University of Economics in Bratislava, Faculty of Economic Informatics, Department of Operations Research and Econometrics, Slovakia

***Abstract:*** *In transport systems, there are usually more distribution companies that must serve their customers. Obviously, if companies could cooperate with each other this kind of cooperation can lead to some additional benefits for them. In this paper, we focus on the situation where we anticipate the existence of multiple companies. Each of these companies owns one central depot whereby their customers are served by different types of vehicles. In the case of non-cooperation behavior, each of the companies act independently. However, if we allow the coalition formation between the companies and consider joint customer service, there may be a reduction in the shipping costs resulting from a better customer allocation to the depot. Other benefits may accrue from the simultaneous use of the depots which means that the vehicle can be repeatedly reloaded at another depot (not the starting one). In this paper, we will introduce the new mathematical models to describe these situation. We will also focus on the possible ways of the redistribution these benefits in terms of Game Theory.*

***Keywords****: Cooperative Game Theory, Game Theory, Vehicle Routing Problem, redistribution*

## 1. INTRODUCTION

This article is dedicated to solve the Vehicle Routing Problem following the Game Theory in terms of cooperative behavior of subjects. We will consider a transport system, whose elements can be characterized as follows: a set of and subjects that realizing customer´s service. Each of this subject (logistic company) owns a depot from which it serves its own set of customers. We consider that the initial customer allocation to the logistic company are known. Each customer requires the delivery of a predetermined quantity of goods. The customer´s service is realized by a vehicle that starts and ends its route in its relevant depot, whereby the type of the vehicle can be different for each logistic company (in terms

* zuzana.cickova@euba.sk

of capacity). We also assume that each point of customer´s service is included in the route of vehicle exactly once, delivery of goods is realized at whole and the capacity of individual depot is not limited. The basic assumption is to achieve the lowest possible transport cost in customer´s service (in the simplest case we minimize the distance between the customers and depots). Concluding on this, it could be modeled as an appropriate Vehicle Routing Problem. Cooperation in Vehicle Routing Problem was solved by many authors. For example, Lin (2008) studied cooperative vehicle routing problem with pickup and delivery time windows ad showed that multiple use of vehicles can reduce costs. Lozano (2013) introduced a mathematical model to quantify the benefits from merging the transport requirements of different companies. For example, McCain (2008) focused on analyzing cooperative games among organizations to increase their profits. It is quite clear, that the cooperation between companies reduces the transport costs and therefore increases the profit of players. This issue was discussed in (Zibaei et al., 2016). The heterogeneous Vehicle Routing Game was solved by Engeval et al. (2004).

Obviously, the shipping costs represent a large amount of the company´s costs of operation and one of the solution to reduce such costs is the cooperation between the logistic companies. Logistic companies create the alliances (coalitions) in which they cooperate with each other in customer´s service. One way to reduce these shipping costs is the better allocation of the customer within a coalition which means that customer can also be served from another depot, not from the central depot. The second way may be the acceptance of the assumption about the return of the vehicle to the starting node, where we assume that the vehicle within a coalition can be repeatedly reloaded at another depot (not the starting one). In this paper we will introduce the new models which describe the situation mentioned above and we will present the basic principle of their functions in the form of illustrative examples.

The formation of a coalition only makes a sense if the coalition behavior of companies provides to the customers (subjects) some kind of surplus (such as a reduction in shipping costs) in comparison to the situation in which the logistic companies acted independently. Obviously, logistic companies will join the coalition in such a case in which they gain some benefits through this approach. Therefore, it is important to determine the redistribution of surplus between the individual players so that the members of individual coalition are not motivated to emerge from the coalition. In terms of the Game Theory, these are the games with a transferable utility (payoff). We will use the Shapley´s value to redistribute the benefits, which is based on the a priori appreciation of each player´s position and strength in terms of the cooperation behavior.

## 2. COOPERATIVE VEHICLE ROUTING PROBLEM WITH HETEROGENEUS FLEET

We will formulate the mathematical model in the full, edged and appreciated graph $\bar{G} = (N, \bar{H})$. Let $N^{(1)} = \{d_1, d_2, \dots d_k\}$ is the set of nodes representing the central depots (centers) and $N^{(2)} = \{c_1, c_2, \dots c_m\}$ is a set of nodes representing the customers and $N = N^{(1)} \cup N^{(2)}$ is the set of all the nodes of the graph. We assume that in every depot is located the one type of vehicle with different capacity and the number of vehicles in the depots is not limited (it can also be interpreted that a car can make a several routes). Capacity of vehicle in individual depots can be labeled as $g_h, h \in N^{(1)}$. We will assume, that vehicles have to return back to its starting point (depot), after they came out from their depots. Let $\bar{H} \subset NxN$ represents the set of edges $e_{ij}$, $i, j \in N$ between all nodes $i$

and *j*. Each edge $e_{ij}$ being assigned to a real number called $d_{ij}$ also known as a price of the edge $e_{ij}$. Let this assignment be the shortest distance between nodes *i* and *j*. Each of the customers located at $i$, $i \in N^{(2)}$, require the import of a certain quantity of goods, generally denominated as $q_i, i \in N^{(2)}$. The goal is to determine the vehicle's routes, which will satisfy the requirements of all customers. Customer requirements will only be realized in the whole (if the vehicle serves the customer, its entire delivery requirement will be realized), with no vehicle capacity exceeded. The main goal is to minimize the total travelled distance. In the model we assume implicitly, that $q_i \leq \max\{g_h, h \in N^{(1)}\}$ for all $i \in N^{(2)}$ which means, that the size of each customer's requirement will not exceed the capacity at least one type of the vehicle.

Now we accept the assumption that the owners of individual depots are different subjects (players). Players are able to cooperate with each other and create the coalitions and reduce their shipping costs. We are considering that each player owns only one depot and also has his own customers, but the player's vehicles in coalition can also be used to serve customers assigned to another player in a possible coalition. The number of possible coalitions is $2^{k+1} - 1$ (where $k+1$ represents the number of players). Let $N_S^{(1)} \subset N^{(1)}$ be a set of coalitions of players. We will also divide a set of customers based on their membership to individual players, then the coalition *S* will customers labeled as $N_S^{(2)}$. Thus, the set $N_S = N_S^{(1)} \cup N_S^{(2)}$ is the set representing the depots and the customers of the coalition *S*.

In general, one way how to mathematically describe routing problems is using binary variables $x_{ijh}$ $i, j \in N, h \in N^{(1)}, i \neq j$ that enable to model if the node *i* precedes node *j* in a route of the vehicle from the *h-th* depot. Further on, the variables $u_{ih}, i \in N^{(2)}, h \in N^{(1)}$ that based on the known Miller-Tucker-Zemlin formulation of Traveling Salesman Problem (1960). Those variables are related to cumulative demand of customers on one particular route.

We assume that vehicle has to return to its starting depot after serving all the customers. The routes of vehicles and new customer assignments can be obtained by using this model:

$$cost = \min f(\mathbf{X}, \mathbf{u}) = \sum_{i \in N} \sum_{\substack{j \in N \\ i \neq j}} \sum_{h \in N^{(1)}} d_{ij} x_{ijh} \tag{1}$$

$$\sum_{i \in N} \sum_{h \in N^{(1)}} x_{ijh} = 1,\ j \in N^{(2)},\ i \neq j \tag{2}$$

$$\sum_{j \in N} x_{ijh} = \sum_{j \in N} x_{jih},\ i \in N^{(2)}, h \in N^{(1)} \tag{3}$$

$$\sum_{j \in N^{(2)}} x_{iji} = \sum_{j \in N^{(2)}} x_{jii},\ i \in N^{(1)} \tag{4}$$

$$\sum_{j \in N^{(2)}} x_{iji} = \sum_{j \in N^{(2)}} \sum_{h \in N^{(1)}} x_{ijh},\ i \in N^{(1)} \tag{5}$$

$$u_{ih} + q_j - g_h\left(1 - x_{ijh}\right) \leq u_{jh},\ i \in N,\ j \in N^{(2)},\ h \in N^{(1)},\ i \neq j \tag{6}$$

$$u_{ii} + q_j - g_i\left(1 - x_{iji}\right) \leq u_{ji},\ i \in N^{(1)},\ j \in N^{(2)},\ i \neq j \tag{7}$$

$$u_{ih} \leq g_h,\ i \in N^{(2)},\ h \in N^{(1)} \tag{8}$$

The scalar cost (1) represents the minimum value of the total travelled distance. Constraint set (2) guarantee that each customer will be visited exactly once and exactly by one vehicle. Conditions (3) a (4) ensure the balance of the route. Constraint set (5) provides the balance of the number of routes from the depot (if this depot is used). Constraints (6) a (7) are the sub-tour elimination conditions and together with the condition (8) ensure that the capacity of the vehicle in not exceeded.

A fundamental assumption for the model presented above is the return of the vehicle to the same center after the completion of the customer´s service. Obviously, the release of this assumption may lead to additional cost savings. Now let´s assume that the vehicles of each depots can be repeatedly reloaded at another depot, not the starting one (while observing the idea of Vehicle Routing Problems). Therefore, we must ensure that the vehicle that is refilled in another depot continues its route to serve customers. Let´s define a new variable:

- $z_{ijh} \geq 0 \; i, j \in N, i \neq j, h \in N^{(1)}$, which represent the order of visit of the edge (*i,j)* by the *h*-th vehicle**.**

We add the constraints to the model mentioned above

$$z_{iji} = x_{iji} \;, \; i, j \in N^{(1)} \tag{9}$$

$$z_{ijh} + 1 \leq z_{jlh} + M(1 - x_{ijh}) + M(1 - x_{jlh}) \;, \; i, j, l \in N, i \neq j, j \neq l, i \neq h, \; h \in N^{(1)}$$
(10)

Where $M$ is a big positive number.

This assumption will allow the possible further cost reductions in the context of cooperative distribution.

## 3. EMPIRICAL RESULTS

Firstly, we are solving our modified cooperative model of Vehicle Routing Problem with heterogenous fleet (1) – (8). The data were obtained from work paper (see at [2]), where we chose the symmetric distance matrix, which respresents the distances of adresses between fifteen customers and three depots in Bratislava. We obtained the shortest distace matrix from origin data by Floyd algorithm.

Consider the net of 18 nodes. We will assume that there are 3 depots from which the vehicle can start its route. So, we consider the distribution problem with multiple depots, whereby we have 3 suppliers to serve the certain customers. Suppliers or players (owners of individual depots) are expressed as $N^1 = \{d_1, d_2, d_3\}$*.* Each player owns one depot with one type of vehicle. The different capacity of each vehicle is given by $g_{d_1}$ = 200*,* $g_{d_2}$ = 220 and $g_{d_3}$ = 230. Customers, who are strictly assigned to the individual depots (players), will be marked as*:* $\{c_1, c_2, c_3, c_4, c_5\} \; for \; d_1$ , $\{c_6, c_7, c_8, c_9, c_{10}\} \; for \; d_2$, $\{c_{11}, c_{12}, c_{13}, c_{14}, c_{15}\} \; for \; d_3$. In the case of the creation the coalition $S \subseteq N^{(1)}$ we know that there are exactly 7 possible coalitions between the players.

We solve the cooperative vehicle routing problem with heterogeous fleet by using the model (1) - (8) and (1)-(10) for the created coalitions *S*: {1}, {2}, {3}, {1,2}, {1,3}, {2,3}, and {1,2,3} by GAMS software. To obtain the optimal solution, we used the solver Cplex 12.2.0.0 on the personal computer INTEL® Core ™ 2 CPU, E8500 @ 3.16 GB RAM for

Windows 10. Our interest is to find optimal solutions by using the model of cooperative vehicle routing problem with heterogeous fleet (1)-(8) ad (1)-(10) and compare their results. Our main interest is to prove that there is a reduction in total shipping costs through mutual cooperation between suppliers.

Table 1. presents the total transport costs of individual coalitions obtained from model (1)-(8) with assumptions mentioned above. In this model, we assume that vehicle has to return to its starting depot after serving all the customers.

Table 1. Minimum transport costs with optimal routes of model (1)-(8)

| Coalitions | Costs | Optimal routes of model (1)-(8) | | | Time processing |
|---|---|---|---|---|---|
| | | Route from $d_1$ | Route from $d_2$ | Route from $d_3$ | |
| S={1} | 22.98 | $d_1$-$c_3$-$c_5$-$c_4$-$c_2$-$c_1$-$d_1$ | | | 0.03 |
| S={2} | 15.32 | | $d_2$-$c_8$-$c_{10}$-$c_9$-$c_7$-$c_6$-$d_2$ | | 0.02 |
| S={3} | 22.7 | | | $d_3$-$c_{13}$-$c_{14}$-$c_{12}$-$c_{11}$-$c_{15}$-$d_3$ | 0.01 |
| S={1,2} | 32.8 | $d_1$-$c_2$-$c_1$-$d_1$ | $d_2$-$c_6$-$c_3$-$c_5$-$d_2$-$c_8$-$c_7$--$c_9$-$c_4$-$c_{10}$-$d_2$ | | 0.97 |
| S={1,3} | 30.78 | $d_1$-$c_2$-$c_{12}$-$c_1$-$c_{11}$-$d_1$ | | $d_3$-$c_{13}$-$c_5$-$c_3$-$c_{14}$-$c_{15}$-$c_4$-$d_3$ | 0.23 |
| S={2,3} | 29.71 | | $d_2$-$c_6$-$c_{14}$-$c_{12}$-$c_{11}$-$c_8$-$d_2$ | $d_3$-$c_{13}$-$c_7$-$c_9$-$c_{15}$-$c_{10}$-$d_3$ | 0.45 |
| S={1,2,3} | 38.25 | $d_1$-$c_2$-$c_{12}$-$c_1$-$c_{11}$-$d_1$ | $d_2$-$c_6$-$c_{14}$-$c_3$-$c_5$-$c_{13}$-$c_7$-$d_2$ | $d_3$-$c_8$-$c_{10}$-$c_{15}$-$c_9$-$c_4$-$d_3$ | 18.88 |

In the Table 2 we present the total transport costs of individual coalitions obtained from model (1)-(10) with two more extra constraints. In this model, we assume that the vehicles of each depots can be repeatedly reloaded at another depot, not the starting one.

Table 2. Minimum transport costs with optimal routes obtained from model (1)-(10)

| Coalitions | Costs | Optimal routes of model (1)-(10) | | | Time processing |
|---|---|---|---|---|---|
| | | Route from $d_1$ | Route from $d_2$ | Route from $d_3$ | |
| S={1} | 22.98 | $d_1$-$c_3$-$c_5$-$c_4$-$c_2$-$c_1$-$d_1$ | | | 0.02 |
| S={2} | 15.32 | | $d_2$-$c_8$-$c_{10}$-$c_9$-$c_7$-$c_6$-$d_2$ | | 0.03 |
| S={3} | 22.7 | | | $d_3$-$c_{13}$-$c_{14}$-$c_{12}$-$c_{11}$-$c_{15}$-$d_3$ | 0.01 |
| S={1,2} | 29.4 | $d_1$-$c_{10}$-$c_4$-$c_9$-$c_7$-$c_8$-$d_2$ | $d_2$-$c_5$-$c_3$-$c_6$-$c_2$-$c_1$-$d_1$ | | 1.11 |
| S={1,3} | 30.38 | $d_1$-$c_{14}$-$c_{12}$-$c_2$-$c_1$-$c_{11}$-$d_1$ | | $d_3$-$c_{13}$-$c_5$-$c_3$-$c_{15}$-$c_4$-$d_3$ | 4.58 |
| S={2,3} | 29.29 | | $d_2$-$c_6$-$c_{14}$-$c_{12}$-$c_{11}$-$c_8$-$d_3$ | $d_3$-$c_{10}$-$c_{15}$-$c_9$-$c_{13}$-$c_7$-$d_2$ | 3.83 |
| S={1,2,3} | 35.21 | $d_1$-$c_{10}$-$c_{15}$-$c_9$-$c_4$-$d_3$ | $d_2$-$c_6$-$c_{12}$-$c_2$-$c_1$-$c_{11}$-$d_1$ | $d_3$-$c_8$-$c_{13}$-$c_7$-$c_5$-$c_3$-$c_{14}$-$d_2$ | 82.39 |

We can summarize various results. For example, if player $d_1$ cooperates with player $d_2$ and player $d_3$, their minimum total cost is 35.21 units (if we consider two extra constraints).

For the comparison, if we obtained the assumptions (9) and (10) to our model, the coalition {1, 2, 3} achieves lower shipping costs. We can also see that after accepting the constraints all amounts of costs are decreasing and we can accept that we obtained much better results.

There are many possible ways of the redistribution these benefits from cooperation in terms of Game Theory. We will use the Shapley´s value, which is based on the a priori appreciation of each player´s position and strength in terms of the cooperation behavior. Firstly, we calculate the cost savings of coalitions $S$ in the case of a cooperative approach to the distribution problem. The cost savings of coalitions $S$ can be calculate as the difference between the sum of player´s total individual costs of noncooperative behavior and the costs, if they cooperate between each other.

Table 3 presents the cost savings, which players can save in case of cooperation. We can also see the redistribution of these savings between the players in coalitions in the next columns. These redistribution is based on Shapley value.

Table 3. Redistribution of benefits by Shapley value

| Coalitions | Costs | Total Individual costs | Costs savings | Redistribution of benefits | | |
|---|---|---|---|---|---|---|
| | | | | Player $d_1$ | Player $d_2$ | Player $d_3$ |
| S={1} | 22.98 | 22.98 | 0 | 0 | | |
| S={2} | 15.32 | 15.32 | 0 | | 0 | |
| S={3} | 22.7 | 22.7 | 0 | | | 0 |
| S={1,2} | 29.4 | 38.3 | 8.9 | 4.45 | 4.45 | |
| S={1,3} | 30.38 | 45.68 | 15.3 | 7.65 | | 7.65 |
| S={2,3} | 29.29 | 38.02 | 8.73 | | 4.365 | 4.365 |
| S={1,2,3} | 35.21 | 61 | 25.79 | 9.72 | 6.435 | 9.635 |

Based on Table 3, we can confirm that in all types of coalitions is sum of the total individual costs of each player higher than the total cost of the coalitions. Therefore, the players tend to cooperate with each other. For this reason, we also quantify the cost savings, which players can save in case of cooperation. It means that in case of cooperation behavior they can save together 25.79 units of costs. If owners of depots act individually, their transport costs are 61 units. Based on Shapley value, we obtain following results of redistribution the benefits. In the coalition $S$= {1, 2, 3} where all owners of depots cooperate with each other, the player $d_1$ saves 9.72 units, the player $d_2$ saves 6.435 units and player $d_3$ saves 9.635 units of shipping costs based on the a priori appreciation of each player´s position and strength in terms of the cooperation behavior.

## 3. CONCLUSION

In this paper, we focused on the cooperative Vehicle Routing Problem with heterogonous fleet assuming the cooperation between the players to minimize the total shipment costs. Our main aim was to compare the results obtained by solving our two models. In the first model (1)-(8), we assume that vehicle has to return to its starting depot after serving all the customers. In the second model (1)-(10), we assume that the vehicles of each depots can be repeatedly reloaded at another depot, not the starting one. Our main idea was to prove that there is a reduction in total transport costs through mutual cooperation between individual suppliers. By comparing our results, we have taken the decision that our modified model has produced much better results than the first model. So, we can state that if we accept the two more constraints about reloading the vehicle at another depot (not the starting one), we obtained better results.

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