MODELLING TRANSPORT ACTIVITIES FROM INVENTORY REPLENISHMENTS IN SUPPLY CHAINS BY USE OF NUMERICAL SIMULATIONS AND MACHINE LEARNING ALGORITHMS

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Abstract: *A periodic review inventory policy represents a standard inventory management model in modern supply chains due to its many advantages. This paper studies its logistic aspects coming from the number and size of transport activities related to inventory replenishments resulting from normally distributed market demand. Due to the stochastic nature of market demand, no simple procedures or algorithms for determining the optimal values of the characteristic variables of the periodic review inventory policy exists, so extensive numerical simulations and symbolic regression analysis of a supply chain echelon are used in this paper. Equations for average order size and required number of orders related to inventory replenishments are developed with R^2 Goodness of Fit and Correlation Coefficient higher than 0.99 tested on 139.500 simulation experiments of a supply chain.*

Keywords*: supply chain management, periodic review, inventories, logistics, symbolic regression*

1. INTRODUCTION

Supply chains (SC) are dynamic systems of high complexity, operating under numerous influential factors. As one of the key elements of supply chain management, the goal of inventory control is to ensure the maximum possible fulfilment of market demand while achieving inventory levels and cost reduction in a highly competitive business environment. As freight transport mainly relies on conventional energy carriers like diesel, kerosene and heavy fuel oil, it significantly contributes to major challenges of the 21st century: pollution and climate change.

This research aims to establish relationships arising from logistical aspects of inventory replenishments in a periodic review inventory policy of a modern supply chain and offer findings to academia and practitioners. Specifically, our research analyses a minimal required number of transportation activities and their size required to fulfil normally distributed market demand for products under varying SCs working conditions. This

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research aims to determine the relations needed for optimisation of transport activities related to inventory replenishments in periodic review inventory policy by the use of advanced techniques of simulation modelling and machine learning. Relevant literature recognises the lack of quantitative models required to examine the possibilities of optimal solutions in terms of operative, economic, and more environmentally suitable approaches. This study aims at contributing to that direction.

1.1. Period review inventory management

Inventory management represents one of the critical processes of supply chain management to balance production and meet the market demand while keeping costs as low as possible. In this context, supply chain managers aim to reach the essential target of an efficient supply chain – having the correct quantity at the right time and in the right place (Longo F., 2011).

A well-known control policy in stochastic inventory control is the (R, s, S) policy, in which inventory is raised to an order-up-to-level S at a review instant R whenever it falls below reorder-level s. Such policies offer several practical advantages and are considered optimal by industry and academia. They facilitate optimal planning and coordination of ordering decisions in multiproduct environments.

One of the essential inventory management strategies is how the company approaches the excess demand and the occurrence of temporary stockouts – it can either be back-ordered or treated as lost sales. Although back-ordering is predominantly present in the relevant literature, according to the research of (Gruen, Corsten, & Bharadwaj, 2002), only 15 % of the customers will, in a real-life setting in an out-of-stock situation, postpone the purchase and wait for the product to be available again. The lost sales environment, which is analysed in this research paper, is prevalent in highly competitive sectors like retail, service, machinery spare parts and online sales (Gruen, Corsten, & Bharadwaj, 2002), (Breugelmans, Campo, & Gijsbrechts, 2006), (Babiloni & Guijarro, 2020). Additionally, the works of (Bashyam & Fu, 1998), (Bijvank, 2014) and (Bijvank & Vis, 2012) indicate that, since the customer satisfaction is often being a differentiation strategy among competitors, and with shortage costs particularly hard to evaluate in practice correctly, service-based requirements are more common in the real-life business sector. As recognised by (Kapalka, Katircioglu, & Puterman, 1999), adding a service-based constraint to an inventory model with lost sales makes the model more realistic but makes computation and analysis more difficult, which results in fewer publications studying this problem. There are no simple procedures or algorithms for determining the optimal values of (R, s, S) characteristic variables in real-world conditions (Babai MZ et al., 2020), (Kiesmüller GP et al., 2011). Consequently, controlling inventories by the subjective assessment, without an algorithmic basis, can result in suboptimal inventory management, increased costs, holding an inadequate amount of inventories and negative consequences such as the bullwhip effect (Žic, S. et al., 2015).

1.2. Logistics

Modern supply chains are recognised as complex systems operating in a global environment, characterised by flexibility in business operations, quick reactions to market demands and changes, the use of technological innovations such as data science, machine learning, artificial intelligence, optimisation of inventories together with the implementation of a green or sustainable approach. Transportation activities are often neglected in the research even though transportation costs can form more than 50% of the total logistics costs of a product (Swenseth S.R. and Godfrey M.R., 2002). Additionally, with uncertainty in the demand patterns, which is the usual case in real-life production and distribution situations, inventory situation gets significantly more complicated (L. Tiacci, S. Saetta, 2009). In logistics, shipment frequency is positively related to fuel consumption and carbon emissions; however, fewer but larger shipments could lead to undesirable higher inventory levels and additional costs (Tang et al., 2015). Unfortunately, many studies (Lee et al. (2005), Van Norden and Van de Velde (2005)) assume that demand data, though variable and non-stationary, must be known. When demand is no longer assumed to be deterministic, as in many production and distribution situations, the introduction of uncertainty in the demand pattern significantly complicates the inventory situation.

In logistics, transportation should provide the most room for cutting carbon emissions since it involves heavy fuel consumption (Andress et al., 2011). The amount of carbon emissions in transportation is determined mainly by transportation frequency and fuel efficiency. Shipment frequency is positively related to fuel consumption as well as carbon emissions. Subsequently, higher-order quantity and less frequent transportation would allow firms to utilise their vehicle capacity better or employ a vehicle with greater transport capacity to save total fuel consumption and reduce carbon emissions. However, fewer but larger shipments lead to undesirable higher inventory levels and additional costs. Therefore, it is natural to ponder whether this approach can reduce emissions effectively and economically and what factors impact the additional cost.

1.3. Machine learning and symbolic regression

Data modelling involves using a limited number of observations of systems variables for inferring relationships among these variables. A number of control parameters characterise the system under study. Empirical modelling attempts to express these critical control variables via other controllable variables that are easier to monitor and can be measured more accurately or timely. Control variables are referred to as outputs. Variables, or properties, used for expressing the response are called inputs or input variables. A combination of values of all input variables and the associated values of the output variables is called a scenario. The modelling task is to detect the driving input variables that cause the change in the observed response variables and formulate the relationship in the form of an accurate model. The quality of this model is assessed by the resemblance of the predicted output to the observed output based on a number of data points. For many industrial applications, the resulting relationships between the input variables and the output variables of a physical system can be found implicitly (Kleijnen, 2005; Kleijnen et al., 2005). Genetic programming (GP) for symbolic regression was first proposed by (Koza 1992) as one of several different applications of genetic programming. Since then, symbolic regression has been widely applied in many engineering sectors, such as industrial data analysis (e.g., Luo et al., 2015; Li et al., 2017), circuits analysis and design (e.g., Ceperic et al., 2014; Shokouhifar & Jalali, 2015; Zarifi et al., 2015), signal processing (e.g., Yang et al., 2005), empirical modelling (e.g., Gusel & Brezocnik, 2011; Mehr & Nourani, 2017), system identification (e.g., Guo & Li, 2012; Wong et al., 2008) and materials analysis (Mu He, Lei Zhang, 2021).

The fact that symbolic regression via GP does not impose any assumptions on the structure of the input-output models means that data determine the model structure. On the other hand, the absence of constraints on the model structure is the greatest challenge for symbolic regression since it vastly increases the search space of possibilities, which is already inherently large. Larger data sets with more input variables and more records make symbolic regression even harder. The rationale of doing the 'evolutionary' search in a vast space of alternatives is to balance the exploitation of the good solutions found so far with exploring the new areas of the search space, where even better solutions may be hiding. At least two or possibly more criteria are used for selecting' good' individuals for further propagation. Often these criteria are prediction error and model expressional complexity. Since these optimisation objectives are competing, the performance of individuals is compared concerning the Pareto-dominance relation in the objective space of model complexity and model error. In Pareto GP, model development happens parallel with automatic identification and exploitation of driving inputs that influence the observed output (Smits et al., 2005). Theoretically, GP can obtain an optimal solution if the computation time is sufficiently long.

2. EXPERIMENT DESIGN

2.1. Market demand simulation

Our model assumes stochastic demand with targeted fill rate as a service-based constraint and the lost sales environment, in which partial deliveries or backlogging is not allowed. Simulation models usually represent a suitable approach when the relations among components do not conform to simple equations or the equation is unknown (Taylor, 2003). Simulation modelling as a tool for analysing various aspects of periodic review inventory policy is present in numerous works.

This study aims to test a wide range of inventory model setup parameters to gather high accountability results. Our research model consists of an inventory simulation model operating under (R, s, S) policy whose output results represent the input information for the symbolic regression model. A general outline of this research is presented in Fig 1.

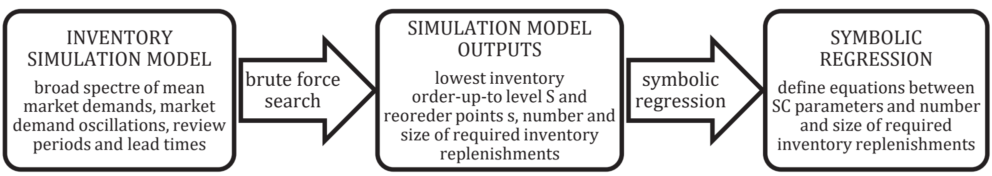

Figure 2. Research framework

The experimental design of our inventory simulation model includes normally distributed market demand with five levels of mean daily demand, three levels of demand oscillations, ten replicas of demand per each combination of mean demand and standard deviation of demand (in total, 150 simulated market demands), 30 levels of the review period, 31 level of lead time and one fill rate of 100% resulting in a total of 139.500 simulation scenarios as presented in Table 1.

Table 1. Inventory model experimental design

Mean market demand (μ)	unit/day	10, 70, 130, 190, 250
Coefficient of variation of market demand (CV)		0.1, 0.2, 0.3
Inventory service performance measures		fill rate (β-service level)
Service levels	%	100
Delivery lead time (LT)	day	0, 1, 2, … 30
Review period (R)	day	1, 2, 3, … 30

The minimal value of CV=0.1 for each value of μ represents the market demand with minimal oscillations. As the oscillations of market demand increase, minimal values of daily market demand get closer to 0, increasing the range between minimal and maximal values of μ to twice the average values. Such high oscillations will result in a significant increase of s and S inventory levels to fulfil the same fill rate compared to the same μ with lower oscillations. Market demand and order size are of non-negative integer values. Market demand is observed for 365 days and can be of value 0 due to the stochastically modelled normally distributed demand. For uniformity of simulation experiments, simulation experiments (SE) start on Monday with inventory levels set to order up to level S. Deliveries executed within the same working day on which the order was launched are referred to as replenishments with zero lead time. Generation of market demands is done in Python and analysis in Prism for Windows, version 9.0.0. Results are visible in table 2, in which each column represents average values of 10 market demands.

Table 2. Descriptive statistics of 150 simulated market demands

Mean	**10**			**70**			**130**			**190**			**250**		
Std. Deviation	**1**	**2**	**3**	**7**	**14**	**21**	**13**	**26**	**39**	**19**	**38**	**57**	**25**	**50**	**75**
Std. Error of Mean	0,1	0,1	0,2	0,4	0,7	1,0	0,7	1,0	2,0	1,0	2,0	3,0	1,0	3,0	4,0
Minimum	7,2	4,4	1,6	49,6	30,3	10,9	92,2	55,7	24,8	130,8	83	27	175,9	107,4	27,7
25% Percentile	9	9	8	65,2	60,2	55,6	121,3	113	103,2	176,9	165,4	152,4	233,3	216,7	198,1
Median	10	10	10	70	70,1	69,5	130	129,9	130	190,3	189,5	189,6	250,1	249,8	250,3
75% Percentile	11	11	12	74,8	79,3	84,2	139	148	156,9	202,8	216	228,1	267,2	283,3	300,6
Maximum	13	15,6	19,2	90,6	109,5	129,5	168,5	207,4	246,8	246,6	308,3	359,9	321,8	399,8	476,5
Range	5,8	11,2	17,6	41	79,2	118,6	76,3	151,7	222	115,8	225,3	332,9	145,9	292,4	448,8
Lower 95% CI of mean	10	10	10	69	69	68	129	127	126	188	186	184	247	245	242
Upper 95% CI of mean	10	10	10	71	71	72	131	133	134	192	194	196	253	255	258
Coefficient of variation	0,1	0,2	0,3	0,1	0,2	0,3	0,1	0,2	0,3	0,1	0,2	0,3	0,1	0,2	0,3
Geometric mean	10	10	9,1	70	69	66	129	127	123,1	189	186	179,9	249	245	236,9
Harmonic mean	10	10	9	69	67	61,4	129	124,1	114	188	181,7	166,2	247	239	217,5
Skewness	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,1	0,0	0,0	0,0	0,0	0,0	0,0
Kurtosis	-0,1	-0,1	0,0	0,0	-0,2	-0,2	0,0	0,0	0,0	0,1	0,1	0,0	-0,1	0,0	0,0

Table 3 shows that all 150 simulated marked demands have passed the normality test according to D'Agostino-Pearson omnibus test from the same Prism software.

Table 3. D'Agostino-Pearson omnibus test of 150 simulated market demands

Mean	10			70			130			190			250		
Std. Deviation	**1**	**2**	**3**	**7**	**14**	**21**	**13**	**26**	**39**	**19**	**38**	**57**	**25**	**50**	**75**
K2	3,62	1,70	0,33	0,47	2,47	1,27	0,18	2,13	1,88	5,04	0,91	0,18	1,97	0,24	2,09
P value	0,16	0,43	0,85	0,79	0,29	0,53	0,91	0,34	0,39	0,08	0,64	0,91	0,37	0,89	0,35
Passed normality test (α=0.05)?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

2.2. Brute force search for lowest characteristic inventory levels s and S

The model uses brute force search to determine the lowest values of s and S, which fulfil the targeted fill rate and satisfy the operating conditions in a total observed period of 365 days. Inventory model outputs represent the basis for the extensive search by symbolic regression method, which results in modelling equations for average order size and required number of inventory replenishments in an observed period.

Fill rate (β) is a quantity-oriented measure, representing the quantity of demand in the observed period which is satisfied from inventory on the stock without back-ordering. This form of service performance measure is relevant for practitioners since it provides a fraction of demand which is turned into sales (Tempelmeier, 2007), (Chopra & Meindl, 2013), (Luo, Bai, Zhang, & Gill, 2014), (Silver, Pyke, & Thomas, 2016). This paper will use fill rate as a service performance measure, calculated according to Eq. 1.

$$\boldsymbol{\beta} = \frac{\boldsymbol{number\ of\ units\ supplied\ from\ the\ stock}}{\boldsymbol{total\ demand}} \tag{1}$$

For this paper, the authors extensively analysed a unique group of β=100%, which is realistic and welcomed in business practice, but theoretical research of such operating conditions of inventory systems is not significantly present. This is so because it was impossible to calculate z value (related to the service level) of normally distributed market demand for which 100% corresponds to an infinitely high value. A schematic view of the data matrix of a simulated supply chain is visible in figure 2.

		Input variables						Responses			
		x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	y_3	y_4
	No	μ	SD	replica	R	LT	β	s	S	average order size	replenishments count
Data records	1	•	•	•	•	•	•	•	•	•	•
	2	•	•	•	•	•	•	•	•	•	•
	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.
	.	.	.	.	.	.	.	.	.	.	.
	139.500	•	•	•	•	•	•	•	•	•	•

Figure 3. Schematic view of data matrix of the simulated supply chain where each line corresponds to a specific scenario; adopted according to (Vladislavleva, 2008)

In order to determine specific s and S values of simulation experiments with the brute force search, it is needed to test Z simulation experiments according to equation 2. Two HP DL580 G8 servers were used for this highly numerically intensive problem. Each server was equipped with 4 Intel Xeon processors with 30 logical cores and 256 GB RAM. In total, $3{,}33 \cdot 10^{12}$ simulation experiments of (R, s, S) inventory control policy were tested to determine 139.500 simulation experiments with the lowest characteristic values for operating conditions. The total computational time for the two above mentioned servers with 120 logical cores each was 4 days, 16 hours and 9 minutes.

$$\boldsymbol{Z = 1 + s - \frac{s - s^2}{2}} \tag{2}$$

Lowest number of required tested simulation experiments, 84 of them were required for μ =10, SD=1, LT=0 and (R, s, S) = (1, 5, 13), which occurred twice. On the other side, the highest number of tested scenarios, 272.732.526 of them, was needed for μ =250, SD=50, LT=30 and (R, s, S) = (30, 16190, 23355), resulting in 3.28 million times more simulation experiments needed for solving one inventory setup with long delivery time, rare review period and high mean values. Results of 139.500 simulation experiments were tested, and characteristic values of (R, s, S) inventory policy were extracted for further analysis. Double-precision values were used since precision, and an adequate number of simulation experiments are of utmost importance for modelling equations by machine learning and symbolic regression processes.

2.3. Modelling responses of (R, s, S) inventory policy with symbolic regression

We used Eureqa demo version 1.24 (Nutonian, 2015) for the modelling part, a software package based on symbolic regression to determine the relationship between the independent and dependent variables. The software searches the fitting parameters and the form of the equations simultaneously (Schmidt and Lipson, 2009). Eureqa's genetic algorithm is multi-objective; two objectives are complexity and error, regardless of the error metric chosen. Eureqa searches for a formula by combining mathematical building blocks, which were: add, subtract, multiply, divide, modulo, square root, min and max. Authors intentionally left out usually used building blocks constants and integer values because they tend to overfit developed models to specific values of input variables and reduce practitioners and academia adaptability of proposed models.

Training and validation data were split equally among 139.500 records and shuffled. As in many industrial and engineering applications recommended, the error metric used was mean squared error (MSE). MSE is the metric that assesses the quality of the forecasting model or predictor. MSE also incorporates both the variance and bias (the distance of predicted value from its actual value). This metric penalises large errors or outliers more than minor errors. Model generation and searches for Pareto optimal equations lasted for 12h on the HP DL580 G8 server for each of the two models. In that time, approximately $3 \cdot 10^{13}$ formulas and $1{,}9 \cdot 10^{6}$ generations were tested.

3. MODELS

The best model selection compromises the minimum acceptable coefficient of determination R^2 and maximum complexity. Equation 3 presents a newly modelled equation for average order size (AOS) for echelon working under normally distributed market demand under (R, s, S) inventory policy and equation 4 required number of inventory replenishments in an observed period N depending on a review period and lead time (RC). It is understood that observed period N must be several times longer than the maximal value of R+LT in order for characteristic inventory values to stabilise. Also, R, LT and N should be in the same time units.

$$\boldsymbol{AOS = \mu\left(R + LT - mod\frac{LT}{R}\right)} \tag{3}$$

$$\boldsymbol{RC = \frac{N}{R+LT-mod\frac{LT}{R}}} \tag{4}$$

As it can be seen from Table 4, statistics of proposed models show high values of R^2 Goodness of Fit and Correlation Coefficient. The equation for average order size shows minimal values for R^2 of 0.997231 and the equation for correlation coefficient of 0.999034. Mentioned values show that proposed equations, albeit simple and tested on an extensive data set of 139.500 simulation experiments, not using integer or decimal constants and not considering SD, are precise even for experimental setups and models.

Table 4. Models statistics

	Average order size	Order count
Mean Absolute Error	52.838	0.420916
Mean Square Error	17663.5	0.626786
R^2 Goodness of Fit	0.997231	0.998439
Correlation Coefficient	0.999034	0.999449
Rank Correlation	0.999	0.996
Maximum Error	1369.83	58
Inter-quartile Absolute Error	7.080	0.338
Signed Difference Error	-43.091	-0.349
Hybrid Correlation Error	0.024	0.022
AIC Error	1.364e+06	-65147.5

Based on the proposed equation, Figure 3 shows an increase in average order size for all tested scenarios based on the mean market demand of 1 product/day. These values can be used for linear scaling to specific mean marked demand in supply chains with periodic review policy. From figure 3 it is visible that if the lead time is multiple of the review period, an increase in average order size will be most significant.

Based on the proposed equation, Figure 4 shows a reduction in the required number of inventory replenishments in an observed period for all tested scenarios. These values are not sensitive to mean market demand or standard deviation of it. From figure 4 it is visible that if the lead time is multiple of the review period, a reduction in the required number of inventory replenishments will be most significant.

4. CONCLUSION

In this paper, authors have analysed (R, s, S) periodic review inventory policy for working conditions of lead time and review period up to 30 days, normally distributed mean market demand between 10 and 250 products daily, CV between 0.1 and 0.3. In total, 139.500 simulation experiments were used for machine learning algorithms to model new equations for average order size and required number of inventory replenishments in an observed period. Proposed equations with values of R^2 Goodness of Fit and Correlation Coefficient higher than 0,99 allow scientists and practitioners to model logistic aspects of the supply chain working under (R, s, S) inventory policy for fulfilling 100% of market demand, reduce SC costs and GHG emissions and optimally plan inventory replenishment activities without reducing the percentage of market demand fulfilment. Proposed equations are simple and do not require the calculation of standard deviation, making them even more useful for supply chain practitioners.

R \ LT	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	60
29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	29	58	58
28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	28	56	56	56
27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	27	54	54	54	54
26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	26	52	52	52	52	52
25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	25	50	50	50	50	50	50
24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	48	48	48	48	48	48	48
23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	46	46	46	46	46	46	46	46
22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	22	44	44	44	44	44	44	44	44	44
21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	42	42	42	42	42	42	42	42	42	42
20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	40	40	40	40	40	40	40	40	40	40	40
19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	38	38	38	38	38	38	38	38	38	38	38	38
18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	36	36	36	36	36	36	36	36	36	36	36	36	36
17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	34	34	34	34	34	34	34	34	34	34	34	34	34	34
16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	32	32	32	32	32	32	32	32	32	32	32	32	32	32	32
15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	45
14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	28	28	28	28	28	28	28	28	28	28	28	28	28	28	42	42	42
13	13	13	13	13	13	13	13	13	13	13	13	13	13	26	26	26	26	26	26	26	26	26	26	26	26	26	39	39	39	39	39
12	12	12	12	12	12	12	12	12	12	12	12	12	24	24	24	24	24	24	24	24	24	24	24	24	36	36	36	36	36	36	36
11	11	11	11	11	11	11	11	11	11	11	11	22	22	22	22	22	22	22	22	22	22	22	33	33	33	33	33	33	33	33	33
10	10	10	10	10	10	10	10	10	10	10	20	20	20	20	20	20	20	20	20	20	30	30	30	30	30	30	30	30	30	30	40
9	9	9	9	9	9	9	9	9	9	18	18	18	18	18	18	18	18	18	18	27	27	27	27	27	27	27	27	27	36	36	36
8	8	8	8	8	8	8	8	8	16	16	16	16	16	16	16	16	24	24	24	24	24	24	24	24	32	32	32	32	32	32	32
7	7	7	7	7	7	7	7	14	14	14	14	14	14	14	21	21	21	21	21	21	21	28	28	28	28	28	28	28	35	35	35
6	6	6	6	6	6	6	12	12	12	12	12	12	18	18	18	18	18	18	24	24	24	24	24	24	30	30	30	30	30	30	36
5	5	5	5	5	5	10	10	10	10	10	15	15	15	15	15	20	20	20	20	20	25	25	25	25	25	30	30	30	30	30	35
4	4	4	4	4	8	8	8	8	12	12	12	12	16	16	16	16	20	20	20	20	24	24	24	24	28	28	28	28	32	32	32
3	3	3	3	6	6	6	9	9	9	12	12	12	15	15	15	18	18	18	21	21	21	24	24	24	27	27	27	30	30	30	33
2	2	2	4	4	6	6	8	8	10	10	12	12	14	14	16	16	18	18	20	20	22	22	24	24	26	26	28	28	30	30	32
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31

Figure 4. Average order size for review period and lead time up to 30 days for mean market demand of 1 product/day under (R, s, S) inventory policy according to Eq. 3

R \ LT	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
30	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	6
29	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	6	6
28	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	7	7	7
27	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	7	7	7	7
26	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	7	7	7	7	7
25	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	7	7	7	7	7	7
24	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	15	8	8	8	8	8	8	8
23	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	16	8	8	8	8	8	8	8	8
22	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	8	8	8	8	8	8	8	8	8
21	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	17	9	9	9	9	9	9	9	9	9	9
20	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	18	9	9	9	9	9	9	9	9	9	9	9
19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	19	10	10	10	10	10	10	10	10	10	10	10	10
18	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	20	10	10	10	10	10	10	10	10	10	10	10	10	10
17	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	21	11	11	11	11	11	11	11	11	11	11	11	11	11	11
16	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	23	11	11	11	11	11	11	11	11	11	11	11	11	11	11	11
15	24	24	24	24	24	24	24	24	24	24	24	24	24	24	24	12	12	12	12	12	12	12	12	12	12	12	12	12	12	12	8
14	26	26	26	26	26	26	26	26	26	26	26	26	26	26	13	13	13	13	13	13	13	13	13	13	13	13	13	13	9	9	9
13	28	28	28	28	28	28	28	28	28	28	28	28	28	14	14	14	14	14	14	14	14	14	14	14	14	14	9	9	9	9	9
12	30	30	30	30	30	30	30	30	30	30	30	30	15	15	15	15	15	15	15	15	15	15	15	15	10	10	10	10	10	10	10
11	33	33	33	33	33	33	33	33	33	33	33	17	17	17	17	17	17	17	17	17	17	17	11	11	11	11	11	11	11	11	11
10	37	37	37	37	37	37	37	37	37	37	18	18	18	18	18	18	18	18	18	18	12	12	12	12	12	12	12	12	12	12	9
9	41	41	41	41	41	41	41	41	41	20	20	20	20	20	20	20	20	20	14	14	14	14	14	14	14	14	14	10	10	10	10
8	46	46	46	46	46	46	46	46	23	23	23	23	23	23	23	23	15	15	15	15	15	15	15	15	11	11	11	11	11	11	11
7	52	52	52	52	52	52	52	26	26	26	26	26	26	26	17	17	17	17	17	17	17	13	13	13	13	13	13	13	10	10	10
6	61	61	61	61	61	61	30	30	30	30	30	30	20	20	20	20	20	20	15	15	15	15	15	15	12	12	12	12	12	12	10
5	73	73	73	73	73	37	37	37	37	37	24	24	24	24	24	18	18	18	18	18	15	15	15	15	15	12	12	12	12	12	10
4	91	91	91	91	46	46	46	46	30	30	30	30	23	23	23	23	18	18	18	18	15	15	15	15	13	13	13	13	11	11	11
3	122	122	122	61	61	61	41	41	41	30	30	30	24	24	24	20	20	20	17	17	17	15	15	15	14	14	14	12	12	12	11
2	183	183	91	91	61	61	46	46	37	37	30	30	26	26	23	23	20	20	18	18	17	17	15	15	14	14	13	13	12	12	11
1	365	183	122	91	73	61	52	46	41	37	33	30	28	26	24	23	21	20	19	18	17	17	16	15	15	14	14	13	13	12	12

Figure 5. Rounded values of the required number of inventory replenishments in an observed period of 365 days for review and lead time up to 30 days under (R, s, S) inventory policy according to Eq. 4

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