

MATHEMATICAL FORMULATION FOR THE CLOSED LOOP INVENTORY ROUTING PROBLEM

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Abstract: *In this paper we observe a closed loop distribution–collection system with a single depot and a set of consumers. Depot has a production facility that produces a single product packed in returnable containers that needs to be delivered to a set of customers. After this product has been used at consumers, returnable containers must be collected and returned to a production facility as an input for the next production cycle. Decision maker must determine, for each day of planning horizon, an exact routing plan for pickup and deliveries in distribution and reverse segment while taking into consideration inventory levels of empty containers and products at depot and consumers. In this paper we present an optimal approach as a mixed integer linear programming model for solving aforementioned closed loop inventory routing problem.*

Keywords: *Inventory routing, Simultaneous pickup and delivery, MILP.*

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1. INTRODUCTION

A closed loop distribution system considers both forward and reverse flows of material. In this paper we observe one-to-many-to-one type of closed loop system with a single depot and a set of consumers. A production facility is located at depot from where a single product is being distributed to a set of consumers. The products are packed in returnable containers. After the products' consumption, empty containers must be returned from consumers to a depot as an input for the next production cycle. For a single time period, this problem can be described as a well known vehicle routing problem with pickup and delivery (VRP-PD), see [1]. Our intention was to extend this problem to multi period planning horizon and to formulate, to the best of our knowledge, a unique closed loop inventory routing problem with simultaneous pickup and delivery with time windows (IRP-SPD-TW). The inventory routing problem (IRP) is another well known research area that simultaneously considers inventories and vehicle routing to optimize delivery plan in a given planning horizon, and recently it has received significant attention from many researchers. An excellent introduction paper to IRP is given by [2], while importance and reasons for the IRP investigation comparing to VRP is given by [3].

Main reason for observing the IRP is the addition of the time dimension in decision making process, comparing to the VRP that in general depends on the space dimension. The most recent IRP survey is given by [4] thirty years after the publication of the first paper ([5]) that simultaneously observed inventories and vehicle routing.

We developed a mixed integer linear programming (MILP) model to have full mathematical understanding of objective function and side constraints of observed IRP-SPD-TW and to be able to benchmark future heuristic approaches that should be developed.

The remaining part of this paper is organized as follows. The paper background with literature overview is given in Section 2. The mathematical formulation is given in Section 3. Section 4 presents test instances and computational results. Finally, some concluding remarks are given in Section 5.

2. BACKGROUND

A growing environmental awareness has lead to reduction of the amount of waste produced and energy consumed in production and distribution systems. These reductions can be achieved by better use of returnable packages, end-of-life and end-of-use products that can be partly or fully disassembled, recycled, remanufactured, and reused. Numerous

legislative regulations among many developed countries demand certain recycling rates, packaging recovery, and active role of manufacturer in the total product lifecycle, which also includes the collection of products. This is the main driving force of reverse logistics, together with costs saving that can be achieved by some kind of product reuse.

Inspiration for our work was the soft drink distribution problem that was observed by [6], and beverage distribution problem that was observed by [7]. In general, bottled drinks (later referred as products) are produced and distributed to a set of consumers. After consumption, empty bottles (later referred as empty containers) must be collected and transported to either recyclable or production facility as an input for next production cycle. Therefore, each consumer has pickup (empty containers) and delivery (products) demands, and this problem is formulated as VRP-PD. Practical application of the VRP-PD can also be found in the case of distribution and collection of books from libraries [8], parcel pickup and delivery service [9], grocery stores replenishment with reusable specialized pallets/containers [10], containership routing [11], home health care [12], printer and photocopier manufacturing industry [13], container drayage [14].

According to [10], in many practical applications costumers that have both pickup and delivery demands want to be serviced with a single stop only, which implies the use of simultaneous pickup and delivery (VRP-SPD) approach. Min [8] was the first author to publish research paper on VRP-SPD.

A prerequisite for IRP is a vendor managed inventory (VMI) system in which supplier makes all decision related to costumers replenishment. As stated by [4], this often leads to a win-win situation where vendors or distributors have more efficient distribution and production, while costumers are freed from inventory control costs. Although the VRP-SPD is intensively being researched, its extension commonly known as inventory routing problem, in this case with simultaneous pickup and delivery (IRP-SPD), is surprisingly unexplored. In the available literature, we didn't found any research papers that consider optimization of both inventories and vehicle routing with simultaneous pickup and delivery over given planning horizon. Additionally, in the practice, pickup and delivery at consumers can only take place on certain time windows, and therefore we observe IRP-SPD-TW.

To the best of our knowledge, four papers were published on the topic of VRP-SPD with time windows (VRP-SPD-TW). The problem observed by [13] is the most similar one to ours, where in general we expanded the problem with a time

dimension, a production facility and an inventory management.

3. MATHEMATICAL FORMULATION

In this paper we observe a closed loop logistics system with one-to-many-to-one distribution of a single product and return flow of that product's empty containers. A multi period planning horizon is considered where in each day production should be maximally utilized and consumers' consumption satisfied. Each consumer must be served by a vehicle within a given time window. A production facility is located at depot which generates products that should be delivered to a set of consumers. Inventory level of products and daily consumer's consumption defines required delivery quantity of products as well as possible generated quantity of empty containers at each consumer per each day of planning horizon. Only available products can be consumed and therefore the quantity of empty containers that can be generated in each day at consumer is defined by available products and its daily consumption. In a production facility, each product is packed in a single empty container. Therefore, inventory level of empty containers and daily production capacity defines the necessary quantity of empty containers that should be collected on daily basis from consumers, and transported to a production facility. Vehicles are located at depot where all routes must start and end their routes. In a single route, vehicle can serve both pickup and delivery demands. Consumers can be served by a single vehicle in a single stop in each day of planning horizon for both pickup and delivery. This implies that the VRP-SPD-TW must be solved for each day of a planning horizon. What will be the pickup quantities and delivery quantities for a production facility and each consumer in each day of planning horizon is the core problem in the IRP-SPD-TW. We assume that each consumer can have a time window in which a vehicle can visit and serve pickup and/or delivery demand.

We use the following notation in the proposed MILP model:

Sets

- I - set of nodes (0 for depot with production facility, 1 and higher for nodes)
- T - set of days in planning horizon
- V - set of vehicles

General parameters

- c_{ij} - travel distance between nodes i and j

- K - vehicles' capacity
 S_i - empty containers inventory level at the beginning of planning horizon in node i
 N_i - product inventory level at the beginning of planning horizon in node i
 P - daily production at depot
 q_i - daily consumption of products in node i
 t_{ser} - node service time
 b_i - start of time window for node i
 e_i - end of time window for node i
 M - big number in objective function
 M_2 - big number in constraints (23) and (24)

Decision variables

- U^t - shortage of empty containers for full productivity of production facility in day t
 x_{vi}^t - quantity of collected empty containers at node i with vehicle v in day t
 h_{vi}^t - quantity of delivered products at node i with vehicle v in day t
 z_i^t - empty containers inventory level at the beginning of day t in node i
 w_i^t - products inventory level at the beginning of day t in node i
 a_i^t - realized consumption at node i in day t (depends on available products)
 r_i^t - product shortage at node i in day t to satisfy daily consumption q_i
 s_{vi}^t - arriving time of vehicle v at node i in day t
 u_{vi}^t - total quantity of empty containers accumulated in vehicle v upon leaving node i
 l_{vi}^t - total quantity of products remaining in vehicle v upon leaving node i
 $y_{vij}^t = \begin{cases} 1 & \text{if vehicle } v \text{ travels directly from node } i \\ & \text{to node } j \text{ in day } t \\ 0 & \text{otherwise} \end{cases}$
 $p_{vi}^t = \begin{cases} 1 & \text{if vehicle } v \text{ pickup empty containers} \\ & \text{from node } i \text{ in day } t \\ 0 & \text{otherwise} \end{cases}$
 $d_{vi}^t = \begin{cases} 1 & \text{if vehicle } v \text{ delivers products to} \\ & \text{node } i \text{ in day } t \\ 0 & \text{otherwise} \end{cases}$
 $f_{vi}^t = \begin{cases} 1 & \text{if vehicle } v \text{ visits node } i \text{ in day } t \\ 0 & \text{otherwise} \end{cases}$

Model formulation presented here primary minimizes shortage of empty containers for full productivity of production facility and shortage of products for full consumption at nodes. Secondly it minimizes vehicles total travel distance. Objective function of the proposed model is defined by (1), subject to constraints (2)-(34).

$$\text{Min} \rightarrow M \cdot \left(\sum_{t=1}^T \left(U^{t+1} + \sum_{i=1}^I r_i^{t+1} \right) \right) + \sum_{t=1}^T \sum_{v=1}^V \sum_{i=0}^I \sum_{j=0}^I c_{ij} \cdot y_{vij}^t \quad (1)$$

Big number M should be large enough so that a single unit of any shortage is more important than savings in vehicles travel distance.

Subject to

$$\sum_{t=1}^T \sum_{v=1}^V x_{v0}^t = 0 \quad (2)$$

$$x_{vi}^t \geq p_{vi}^t \geq \frac{1}{K} \cdot x_{vi}^t \quad \forall t \in T, \forall v \in V, \forall i \in I, i > 0 \quad (3)$$

$$h_{vi}^t \geq d_{vi}^t \geq \frac{1}{K} \cdot h_{vi}^t \quad \forall t \in T, \forall v \in V, \forall i \in I, i > 0 \quad (4)$$

$$p_{vi}^t + d_{vi}^t \geq f_{vi}^t \geq \frac{(p_{vi}^t + d_{vi}^t)}{2} \quad \forall t \in T, \forall v \in V, \forall i \in I \quad (5)$$

Constraints (2) forbid empty containers from production facility. Constraints (3) defines if vehicles v pickup empty containers from node i in day t . Constraints (4) defines if vehicles v delivers products to node i in day t . Constraints (5) defines if vehicles v visits node i in day t .

Inventory related constraints

$$z_i^1 = S_i \quad \forall i \in I \quad (6)$$

$$w_i^1 = N_i \quad \forall i \in I \quad (7)$$

$$z_0^{t+1} = z_0^t - P + \sum_{v=1}^V \sum_{i=1}^I x_{vi}^t \quad \forall t \in T \quad (8)$$

$$w_0^{t+1} = w_0^t + P - U^t - \sum_{v=1}^V \sum_{i=1}^I h_{vi}^t \quad \forall t \in T \quad (9)$$

The starting inventory level of empty containers and products at the beginning of planning horizon in all nodes are set by constraints (5), (6). The inventory level of empty containers and products at the end of day t and beginning of day $t+1$ in production facility is defined by constraints (8), (9).

$$a_i^1 = q_i \quad \forall i \in I, i > 0 \quad (10)$$

$$a_i^{t+1} = q_i - r_i^{t+1} \quad \forall t \in T, \forall i \in I, i > 0 \quad (11)$$

$$r_i^{t+1} \geq q_i - w_i^{t+1} \quad \forall t \in T, \forall i \in I, i > 0 \quad (12)$$

We set realized consumption to full intensity for the first day of planning horizon by constraints (10). For each following day, by constraints (11) we define the realized consumption for each node.

Inventory shortage of products at each node is defined by constraints (12).

$$z_i^{t+1} = z_i^t + a_i^t - \sum_{v=1}^V x_{vi}^t \quad \forall t \in T, \forall i \in I, i > 0 \quad (13)$$

$$w_i^{t+1} = w_i^t - a_i^t + \sum_{v=1}^V h_{vi}^t \quad \forall t \in T, \forall i \in I, i > 0 \quad (14)$$

The inventory level of empty containers and products at the end of day t and beginning of day $t+1$ in each node are defined by constraints (13), (14).

$$U^{t+1} \geq 2 \cdot P - z_0^t - \sum_{v=1}^V \sum_{i=1}^I x_{vi}^t \quad \forall t \in T \quad (15)$$

Quantity of empty container shortage at production facility per each day of planning horizon is defined by constraints (15).

Routing related constraints

$$\sum_{v=1}^V f_{vi}^t \leq 1 \quad \forall t \in T, \forall i \in I \quad (16)$$

$$\sum_{v=1}^V x_{vi}^t \leq z_i^t \quad \forall i \in I, i > 0 \quad (17)$$

$$x_{vi}^t \leq K \cdot p_{vi}^t \quad \forall t \in T, \forall v \in V, \forall i \in I \quad (18)$$

$$\sum_{i=1}^I \sum_{v=1}^V h_{vi}^t \leq w_0^t \quad \forall t \in T \quad (19)$$

$$h_{vi}^t \leq K \cdot d_{vi}^t \quad \forall t \in T, \forall v \in V, \forall i \in I \quad (20)$$

Constraints (16) define that one node can be visited by only one vehicle per each day. Constraints (17) defines that empty containers can be picked up from node only if that node is in pickup plan and limits the value of pickup quantity in node i to available empty containers inventories. Constraints (18) limits the value of pickup quantity in node i to vehicles capacity. Constraints (19) define the maximal products quantities that can be delivered from depot to all nodes in a single day. Constraint (20) defines that products can be delivered to node only if that node is in delivery plan and that quantity is limited to vehicles capacity.

$$\sum_{j=1}^I y_{vij}^t = f_{vi}^t \quad \forall t \in T, \forall v \in V, \forall i \in I, i \neq j \quad (21)$$

$$\sum_{j=1}^I y_{vji}^t = f_{vi}^t \quad \forall t \in T, \forall v \in V, \forall i \in I, i \neq j \quad (22)$$

$$b_i \cdot f_{vi}^t \leq s_{vi}^t \leq e_i + M_2 \cdot (1 - f_{vi}^t) \quad \forall t \in T, \forall v \in V, \forall i \in I \quad (23)$$

$$s_{vi}^t + y_{vij}^t \cdot (d_{ij} + t_{ser}) \leq s_{vj}^t + M_2 \cdot (1 - y_{vij}^t) \quad \forall t \in T, \forall v \in V, \forall i \in I, \forall j \in I, i \neq j, i > 0, j > 0 \quad (24)$$

Constraints (21) define that sum of all incoming arc to visited node must be equal to one and constraints (22) define that sum of all outgoing arc from visited node also must be equal to one; if a node is not being visited by any vehicle these sums are equal to zero. Vehicles arriving time at nodes must be in given time windows for each node, which is defined by constraints (23). Constraints (24) ensure that a vehicle time of arrival at successor node has greater value than arrival time at predecessor node in one route. Also, these constraints eliminate subtours for each route. Big number M_2 should be large enough so that constraints (24) are always valid for cases when a vehicle does not travel from node i to node j . This can be achieved if M_2 is set to maximal working time (we assume that vehicles speed is one minute per unit of distance).

$$u_{v0}^t = 0 \quad \forall t \in T, \forall v \in V \quad (25)$$

$$l_{v0}^t = \sum_{i=1}^I h_{vi}^t \quad \forall t \in T, \forall v \in V \quad (26)$$

$$0 \leq u_{vi}^t + l_{vi}^t \leq K \quad \forall t \in T, \forall v \in V, \forall i \in I \quad (27)$$

$$u_{vi}^t + x_{vj}^t + (1 - y_{vij}^t) \cdot K \geq u_{vj}^t \geq u_{vi}^t + x_{vj}^t - (1 - y_{vij}^t) \cdot K \quad \forall t \in T, \forall v \in V, \forall i \in I, \forall j \in I, i \neq j, j > 0 \quad (28)$$

$$l_{vi}^t - h_{vj}^t + (1 - y_{vij}^t) \cdot K \geq l_{vj}^t \geq l_{vi}^t - h_{vj}^t - (1 - y_{vij}^t) \cdot K \quad \forall t \in T, \forall v \in V, \forall i \in I, \forall j \in I, i \neq j, j > 0 \quad (29)$$

Total quantity of empty containers in vehicle v upon leaving depot is set to zero by constraints (25) and total quantity of products remaining in vehicle v upon leaving depot is set by constraints (26) to sum of all delivery quantities for vehicle v in observed day. Constraints (27) defines the minimal and maximal quantities of products and empty containers in vehicle upon leaving node i . Constraints (28) defines the quantity of empty containers in vehicle upon leaving node j . Constraints (29) defines the quantity of products in vehicle upon leaving node j .

$$U^{t+1} \geq 0 \quad \forall t \in T \quad (30)$$

$$z_i^{t+1}, w_i^{t+1}, r_i^{t+1}, a_i^{t+1} \geq 0 \quad \forall t \in T, \forall i \in I \quad (31)$$

$$x_{vi}^t, h_{vi}^t, u_{vi}^t, l_{vi}^t, s_{vi}^t \geq 0 \quad \forall t \in T, \forall v \in V, \forall i \in I \quad (32)$$

$$p_{vi}^t, d_{vi}^t, f_{vi}^t \in \{0,1\} \quad \forall t \in T, \forall v \in V, \forall i \in I \quad (33)$$

$$y_{vij}^t \in \{0,1\} \quad \forall t \in T, \forall v \in V, \forall i \in I, \forall j \in I, i \neq j \quad (34)$$

Constraints (30)-(32) define decision variables that take positive integer values, and constraints (33) and (34) define the binary nature of decision variables.

4. COMPUTATIONAL RESULTS

In order to test the MILP model we have randomly generated 10 test instances with the following parameters: one production facility and 10 nodes ($I=10$); 2 vehicles that can transport up to 15 units ($K=15$); node consumption can have any integer value from [1, 4]; 60 min node's time window length can begin at any full hour between [60 min, 360 min]; planning horizon of 4 days ($T=4$); the spatial coordinates of nodes are randomly generated as integers in a square [-50, 50] units and the location of the depot is in the center of that square (coordinates (0, 0)); travel distance between nodes i and j is calculated as Euclidian distance; productivity of production facility is set to $P=25$; inventory level in production facility at the beginning of planning horizon is randomly generated as integers between $[P, P + P/5]$ for both products and empty containers; inventory level in each node at the beginning of planning horizon is randomly generated as integers between $[q_i, q_i*1.5]$, with additional condition that sum of all node's beginning inventory levels must be between P and $P+P/10$ (for both products and empty containers); node service time is set to $t_{ser}=10$; maximum working time is set to 8 hours ($b_i=0$ min, $e_i=480$ min); big number in objective function is set to $M=100$; big number in constraints is set to $M_2=480$.

Table 1 shows input parameters for instance 1. The solution for instance 1, represented in deliveries and pick-ups at nodes in planning horizon, is given in Table 2. The solution routes for instance 1 is presented in Figure 1.

The results for all 10 test instances are presented in Table 3 which contains: objective function values; sum of all empty containers shortages for full productivity of production facility in planning horizon; sum of all product shortages at nodes in planning horizon; and CPU time.

Table 1. Input parameters for instance 1

| i | x_i | y_i | S_i | N_i | q_i | b_i | e_i |
|-----|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 29 | 27 | - | 0 | 480 |
| 1 | -29 | 43 | 2 | 2 | 2 | 360 | 420 |
| 2 | -19 | 31 | 4 | 3 | 3 | 180 | 240 |
| 3 | 30 | -18 | 2 | 3 | 2 | 180 | 240 |
| 4 | 21 | -31 | 3 | 3 | 3 | 300 | 360 |
| 5 | -3 | -38 | 3 | 3 | 3 | 360 | 420 |
| 6 | 11 | 1 | 2 | 2 | 2 | 360 | 420 |
| 7 | 18 | 47 | 1 | 1 | 1 | 300 | 360 |
| 8 | -45 | 15 | 1 | 1 | 1 | 300 | 360 |
| 9 | 9 | 34 | 4 | 5 | 4 | 300 | 360 |
| 10 | -5 | -43 | 4 | 4 | 4 | 360 | 420 |

Table 2. Solution for instance 1: deliveries and pick-ups at nodes in planning horizon

| i | $t=1$ | | $t=2$ | | $t=3$ | | $t=4$ | |
|-------|--------|-------|--------|-------|--------|-------|--------|-------|
| | deliv. | pick. | deliv. | pick. | deliv. | pick. | deliv. | pick. |
| 1 | - | - | 6 | 4 | - | - | - | - |
| 2 | 3 | 4 | 3 | 3 | 5 | 2 | 1 | 4 |
| 3 | 2 | 2 | 2 | 1 | 2 | 2 | 2 | 2 |
| 4 | 6 | 3 | - | - | 3 | 6 | 3 | 3 |
| 5 | 3 | 1 | 3 | 5 | 3 | 3 | 3 | 3 |
| 6 | 4 | 2 | - | - | 4 | 4 | - | 2 |
| 7 | 2 | 1 | 1 | 1 | - | - | 1 | 2 |
| 8 | - | - | 3 | 2 | - | - | - | - |
| 9 | 3 | 4 | 4 | 5 | 4 | 4 | 4 | 4 |
| 10 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 |
| Total | 27 | 21 | 26 | 25 | 25 | 25 | 18 | 24 |

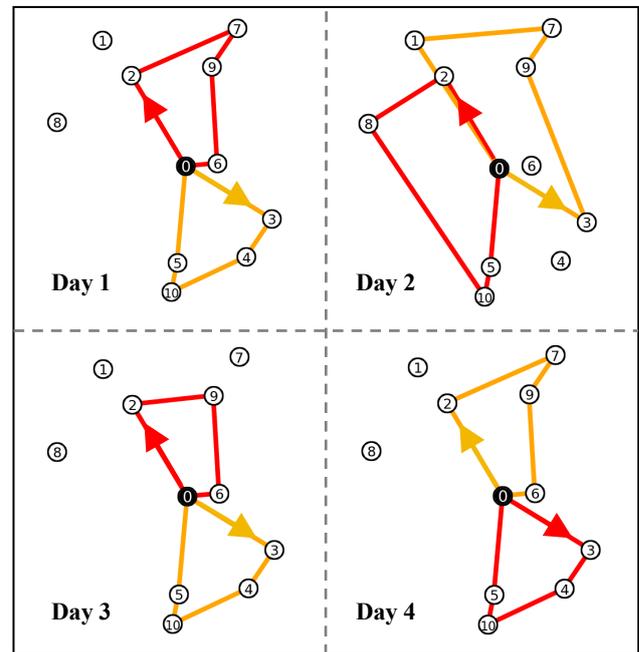


Figure 1. Obtained routes per each day for instance 1

MILP model was implemented by the CPLEX 12.2 on the Intel(R) Core(TM) i3 CPU M380 2.53 GHz with 6 GB RAM.

5. CONCLUSION

In this paper we developed the MILP model for solving the closed loop inventory routing problem

with simultaneous pickup and delivery with time windows with objective to provide continuous supply of the production facility and consumers under minimal transport costs. The MILP model was able to solve 9 out of 10 small scale instances. Instance 2 could not be solved in maximum allowed CPU time of 1800 sec. This implies that the MILP model is sensitive to instance input parameters. Additionally, CPU time for solving the small scale problems indicate that larger problems could not be solved to optimality in a reasonable time.

Table 3. Results from the MILP model

| Instance | Obj. func. | ΣU^{+1} | $\Sigma \Sigma r^{+1}_i$ | CPU time [sec] |
|----------|------------|-----------------|--------------------------|----------------|
| 1 | 1559 | 1 | 3 | 29.6 |
| 2 | 2077 | 3 | 4 | *1800.0 |
| 3 | 1292 | 0 | 0 | 27.1 |
| 4 | 1419 | 0 | 1 | 101.0 |
| 5 | 1579 | 0 | 0 | 57.3 |
| 6 | 1850 | 0 | 1 | 88.6 |
| 7 | 1222 | 0 | 1 | 239.0 |
| 8 | 1832 | 0 | 1 | 219.0 |
| 9 | 1598 | 0 | 0 | 401.8 |
| 10 | 1505 | 0 | 0 | 99.9 |
| Avg. | 1593.3 | 0.4 | 1.1 | 306.3 |

* Solution obtained after 1800 sec of CPU time (CPLEX parameter *timelimit* is set to 1800).

Therefore, future research should include comprehensive model testing regarding characteristics of vehicle fleet and time windows as well as development of a heuristic approach for solving larger scale problem instances that are more realistic.

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