COST ANALYSIS OF OPEN VRP

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Abstract: *Classical routing problems aim at designing the minimum cost of routes originating and finishing from a central depot for satisfying customer demand. The open versions do not consider the vehicles need to be returned. The importance of open versions of the known routing problems arises e.g. from car-hire options. Although hiring vehicles could be more expensive per unit distance traveled, their use could lead to considerable saving due to reduction of the total cost of the route. Open models of routing problems allow describing different real-world applications, e.g. considering one or more vehicles of homogenous or heterogeneous fleet structure (own or leased) starting in one of different depots that result to different logistic costs. The paper is focused on cost analysis that may aid in the decision to use own and hired vehicles based on regional data in Slovakia.*

Keywords: *Open Vehicle Routing Problem, Mixed Integer Programming, Mathematical Model.*

1. INTRODUCTION

Logistics costs constitute a major share of the total costs of almost every organization. Many variants of routing problems can be very rewarding in the field of logistics. The well-known capacitated vehicle routing problem (CVRP) is one of the most discussed problems in operations research Čičková and Brezina (2008), Čičková et al. (2013), Čičková et al. (2014), Desrochers et al. (1992), Fábry et al. (2011). The significance of that problem is evidently depended of its computational complexity, but also the importance follows from a great practical applicability, thus the problem can be applied in more general way.

This paper describes, based on regional data in Slovakia, the version of open vehicle routing problem (OVRP) that allows combining CVRP together with the possibility to realize open routes, e.g. Čičková et al. (2014). In general, OVRP is a popular problem in the field of distribution management and can be used to model many real-life problems. For example if the company deals without its own vehicle fleet and has to hire some vehicle to deliver its products to customers, it is not concerned whether the vehicle returns to the depot, and does not pay the traveling costs between the last customer and the depot. The standard OVRP can be described as follows: Consider a depot from which some products have to be delivered to a set of customers. Products are loaded on a vehicle (vehicles) at the depot and afterwards load needs to be transported to the customers. The capacity of vehicle (or fleet of vehicles) is (are) known (if more than one vehicle are used, the same capacity of all of them is supposed), so that all customers' demand needs to be served with the use of some vehicle (all the demands are met in full). It is assuming the known shortest distance between depot and each customer's location, as well as between each pairs of customer's location. The goal is to find the optimal shortest route for a vehicle (vehicles) so that each customer demand is met.

Practical applications often consider the deal to specify the number of own vehicles relative to cost of rented vehicle. The unit cost per km of own vehicles consist not only of fuel cost but also include e.g. amortization, taxes and driver's wage. On the other side, the rental cost depends on cost per km and also includes the fixed cost per car. Presented analysis is based on example of distribution in regions in Slovakia. The practical significance lies in modeling centrally controlled logistic projects that consist of two stages (e.g. organizing of vote distribution). The first stage consists in transferring elements (e.g. ballots) to regional capitals and their further distribution to relevant district towns.

2. MATHEMATICAL MODEL

The mathematical formulation based on above mentioned can be stated as follows: Consider graph $G = (N_0, A)$, where $N_0 = \{0, 1,\ldots n\}$ is the set of all nodes in the graph, so that $N_0 = N \cup \{0\}$, where the set $N = \{1, 2\ldots n\}$ represents the set of served nodes (customers) and the node indexed 0 represents origin (depot). The set $A = \{(i, j): i, j \in N_0, i \neq j\}$ is the arc set of G. A shortes distance d_{ij} is associated with every arc of the graph. Let the parameters g_i, $i \in N$ represent demand of customer and parameter g represents the capacity of vehicles. Further on, the cost (per km) of own vehicles (co) and also the cost (per km) of rented vehicles (cr) are known. The fixed cost are asociated only with number of rented vehicle and they are designated as cf. Mathematical programming formulation requires two type of binary variables: the variables x_{ij}, $i, j \in N_0$ with a following notation: $x_{ij} = 1$ if customer i precedes customer j in a route of the own vehicle and $x_{ij} = 0$ otherwise and the variables y_{ij}, $i, j \in N_0$ with a following notation: $y_{ij} = 1$ if customer i precedes customer j in a route of the rented vehicle and $y_{ij} = 0$ otherwise. Further on, we will apply the variables u_i, $i \in N$ that based on well-known Miller –Tucker- Zemlin's formulation, e.g. Miller et al. (1960), of the traveling salesman problem. That variables will represent cumulative demand of customers on one particular route.

The mathematical model can be stated as follows:

$$\min f(\mathbf{X},\mathbf{u}) = co\sum_{i\in N_0}\sum_{\substack{j\in N_0\\ i\neq j}} d_{ij}x_{ij} + cr\sum_{i\in N_0}\sum_{\substack{j\in N_0\\ i\neq j}} d_{ij}y_{ij} + cf\sum_{j\in N_0} y_{0j} \tag{1}$$

$$\sum_{i\in N_0} x_{ij} + \sum_{i\in N_0} y_{ij} = 1,\ j\in N,\ i\neq j \tag{2}$$

$$\sum_{j\in N_0} x_{ij} + \sum_{j\in N_0} y_{ij} \leq 1,\ i\in N,\ i\neq j \tag{3}$$

$$u_i + q_j - g(1-x_{ij}) \leq u_j,\ i\in N_0,\ j\in N,\ i\neq j \tag{4}$$

$$u_i + q_j - g(1-y_{ij}) \leq u_j,\ i\in N_0,\ j\in N,\ i\neq j \tag{5}$$

$$\sum_{i\in N_0} x_{ij} - \sum_{i\in N_0} x_{ji} = 0,\ j\in N,\ i\neq j \tag{6}$$

$$q_i \leq u_i \leq g,\ i\in N \tag{7}$$

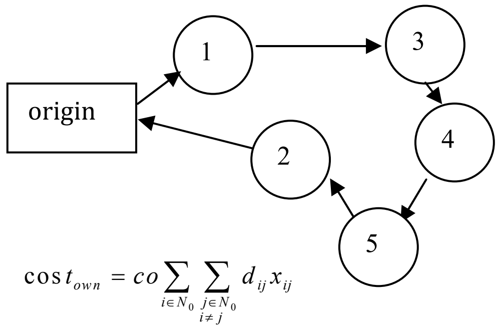
$$u_0 = 0 \tag{8}$$

$$x_{ij}\in\{0,\ 1\},\ y_{ij}\in\{0,\ 1\},\ i,j\in N_0,\ i\neq j \tag{9}$$

The objective function (1) models the total cost for all the used vehicles. Equations (2) ensure that only one of the vehicles (own or rented) enters each customer exactly once and equations (3) ensure that the vehicle does not need to depart from every customer, because the route of rented vehicle ends after serving the last of them. Equations (5) and (6) avoid the presence of sub-tour (for own and rented vehicle) and also calculates the real cumulative demands of customers of the next node on the route based on previous node. Equations (7) ensure that all demands on the route must not exceed the capacity of the vehicle. The fix values of variables u_0 are set up by equation (8).

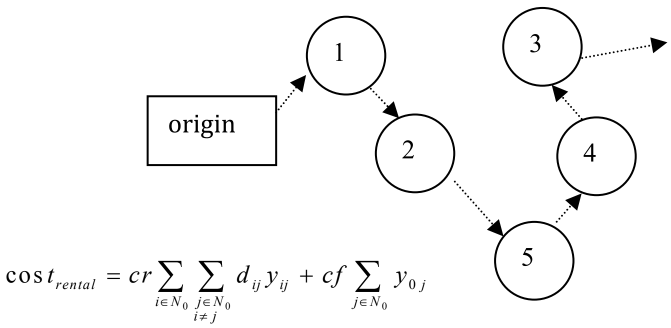
3. OPEN VEHICLE ROUTING PROBLEM IN SLOVAK REGIONS

The problem deals about the distribution scheduling in the regions in Slovakia, where origins were situated in the regional capitals. The distribution have to be assured using the own or rented vehicles with the same capacity. The goal was to determine how many own and rented vehicles must be used in each region so that demand of all district towns must be met with the minimal cost. The total cost consist of cost associated with the own vehicles (Fig. 1a) and cost of rented (Fig. 1b).



$$\cos t_{own} = co \sum_{i \in N_0} \sum_{\substack{j \in N_0 \\ i \neq j}} d_{ij} x_{ij}$$

Fig. 1a



$$\cos t_{rental} = cr \sum_{i \in N_0} \sum_{\substack{j \in N_0 \\ i \neq j}} d_{ij} y_{ij} + cf \sum_{j \in N_0} y_{0j}$$

Fig. 1b

Slovakia is divided into 8 regions: Region Bratislava (BA), Region Banská Bystrica (BB), Region Košice (KE), Region Nitra (NR), Region Prešov (PO), Region Trenčín (TN), Region Trnava (TT) and Region Žilina (ZA):

Region Bratislava (BA) is divided into 4 districts: Bratislava (0), Malacky (1), Pezinok (2), Senec (3).

Region Banská Bystrica (BB) is divided into 13 districts: Banská Bystrica (0), Banská Štiavnica (1), Brezno (2), Lučenec (3), Detva (4), Krupina (5), Poltár (6), Revúca (7), Rimavská Sobota (8), Veľký Krtíš (9), Zvolen (10), Žarnovica (11), Žiar nad Hronom (12).

Region Košice (KE) is divided into 7 districts: Košice (0), Gelnica (1), Michalovce (2), Rožňava (3), Sobrance (4), Spišská Nová Ves (5), Trebišov (6).

Region Nitra (NR) is divided into 7 districts: Nitra (0), Komárno (1), Levice (2), Nové Zámky (3), Šaľa (4), Topoľčany (5), Zlaté Moravce (6).

Region Prešov (PO) is divided into 13 districts: Prešov (0), Bardejov (1), Humenné (2), Kežmarok (3), Levoča (4), Medzilaborce (5), Poprad (6), Sabinov (7), Snina (8), Stará Ľubovňa (9), Stropkov (10), Svidník (11), Vranov nad Topľou (12).

Region Trenčín (TN) is divided into 9 districts: Trenčín (0), Bánovce nad Bebravou (1), Ilava (2), Myjava (3), Nové Mesto nad Váhom (4), Partizánske (5), Považská Bystrica (6), Prievidza (7), Púchov (8).

Region Trnava (TT) is divided into 7 districts: Trnava (0), Dunajská Streda (1), Galanta (2), Hlohovec (3), Piešťany (4), Senica (5), Skalica (6).

Region Žilina (ZA) is divided into 11 districts: Žilina (0), Bytča (1), Čadca (2), Dolný Kubín (3), Kysucké Nové Mesto (4), Liptovský Mikuláš (5), Martin (6), Námestovo (7), Ružomberok (8), Turčianske Teplice (9), Tvrdošín (10).

Input data: 8 regions (8 problems solved), number of delivery district towns and regional capital in each region (BA – 4, BB – 13, KE – 6, NR – 7, PO – 13, TN – 9, TT – 7, ZA – 11), distribution centre (regional capital) in each problem is indexed as $i = 0$, the shortest distances between all district towns and between distribution centre and each district towns is designated as c_{ij}, vehicle capacity V was set to 12, the demand of district towns was associated with number of inhabitant (0.00006 m^3 per inhabitant), cost per km of own vehicle was set to co = 0.5 €, cost for rented vehicle was set as follows: fixed cost per route cf = 15 €, cost per km cr = 0.6 €. The goal was to minimize the total costs for the distribution in each region and to determine the number of own and rented vehicles, with respect the following restrictions: the origin 0 is the initial node and also the final node for each own vehicle route, the final point in case of rented vehicle is the last served node.

The computational experiments were provided on the base of before mentioned data. The mathematical model was implemented in software GAMS (solver Cplex 12.2.0.0) on PC with Intel ® Core ™ i7-3770 CPU with a frequency of 3.40 GHz and 8 GB of RAM under MS Windows 8. The results are shown in Table 1a, 1b .

Table 1a Regional distribution

	Region Bratislava	Region Banská Bystrica	Region Košice	Region Nitra
Route 1	0-1-2-3-0	0-10-4-1-11-12-0	0-1-5-3	0-3-0
Route 2		0-2-7-8	0-2-4	0-5-0
Route 3		0-3-6-5-9	0-6	0-6-2-0
Route 4				0-4-1
Total cost	52	261.7	186	165
Cost for own vehicles	52	68.5	0	102
Total cost for rented vehicles	0	193.2	186	63
Fix cost for rented vehicles	0	30	45	15
Variable cost for rented vehicles	0	163.2	141	48

Table 1b Regional distribution

	Region Prešov	Region Trenčín	Region Trnava	Region Žilina
Route 1	0-7-0	0-1-3-4-0	0-2-0	0-1-4-2-0
Route 2	0-4-6-3	0-2-8-6-0	0-4-3-0	0-6-9-5
Route 3	0-9-1-11-10-5	0-5-7	0-1	0-8-3-10-7
Route 4	0-12-2-8		0-5-6	
Total cost	255.2	159.1	158.1	198.7
Cost for own vehicles	14	104.5	58.5	38.5
Total cost for rented vehicles	241.2	54.6	99.6	160.2
Fix cost for rented vehicles	45	15	30	30
Variable cost for rented vehicles	196.2	39.6	69.6	130.2

The Table 1 provides the solution in each region in Slovakia. The first 4 rows of the table show the obtained routes in region (if last node is of the route is the node indexed 0 than the route is realized by own vehicle and otherwise by rented vehicle) and 5th row shows the total cost of distribution (objective function) in selected region. The distribution system is depicted on Fig.2, where the solid lines represent the routes of own vehicles and dotted lines represent the routes of rented vehicles.

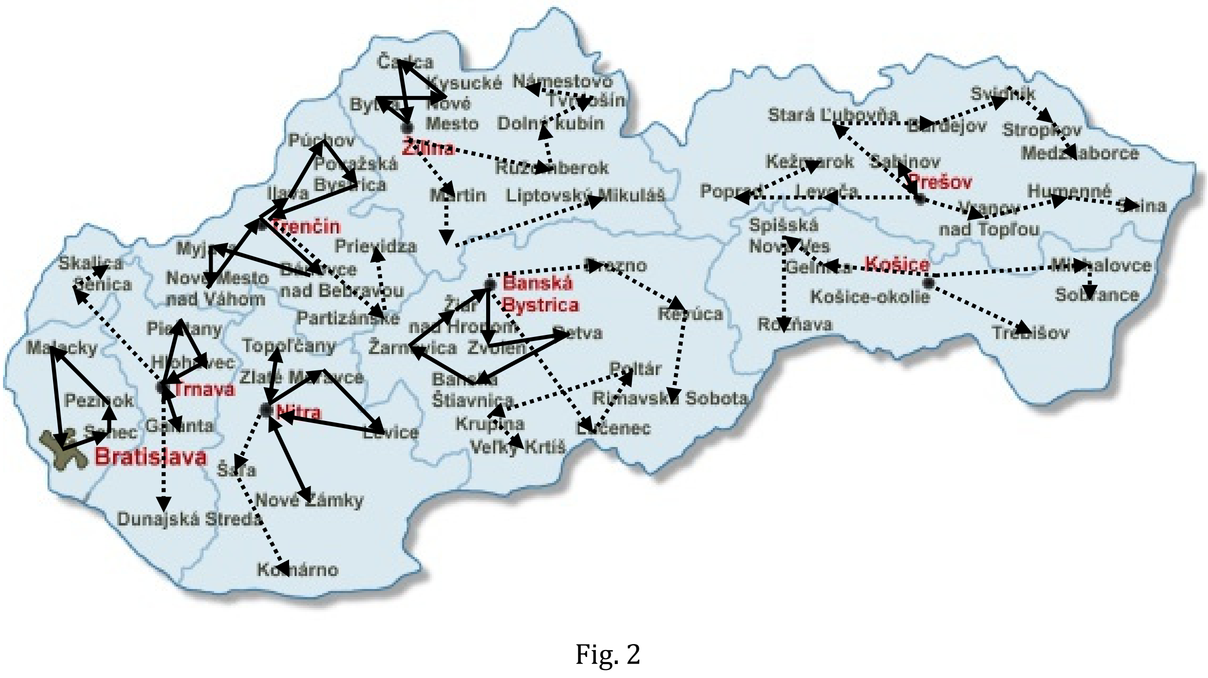


Fig. 2

4. CONCLUSION

This paper considers the version of open vehicle routing problem (OVRP) that allows combining CVRP together with the possibilities of open routes. The mathematical formulation was provided on the base of mixed integer programming (MIP) with linear objective function and constraints. So that formulation allows the use of standard software for solving MIP problems. The computational experiments were based on regional data in Slovakia. The distribution has to he assured from regional capital to district towns in each region using the own or rented vehicles. The goal was to determine how many own and rented vehicles must be used in each region so that demand of all district towns must be met with the minimal cost (the total cost consist of the cost of own vehicles and of the cost of rental). Software implementation was realized in GAMS (solver Cplex).

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