NEW MATHEMATICAL FORMULATIONS OF THE DYNAMIC BERTH ALLOCATION PROBLEM

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Abstract: *This paper considers Dynamic Berth Allocation Problem. The mathematical formulation of this problem was proposed by Imai et al. (2001). Trying to apply proposed mix-integer programming model we have noticed that the model has some weaknesses. In order to mitigate the observed drawbacks we proposed changes in the original mathematical formulation. We tested the models on the small size example.*

***Keywords**: Dynamic Berth Allocation Problem, mathematical formulation, mix-integer programming.*

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1. INTRODUCTION

The Berth Allocation Problem represents one of the most important port optimization problems. This problem is also known in the literature as the berth scheduling problem. The Berth Allocation Problem concerns the allocation of berth space for vessels. The dispatchers in terminals have to assign, as soon as possible, arriving vessels to berth to be loaded and/or unloaded. There are various types of the Berth Allocation Problem, depending on the berthing space (discrete or continuous), the portion of the vessels to be assigned that are already in the port (static or dynamic Berth Allocation Problem), the nature of vessel handling time (handling time is an input, or handling times are decision variables), existence of service priorities, etc.

Lai and Shih 1992 proposed three allocation policies and compared them with allocation current allocation policies in Hong Kong. To evaluate proposed allocation policies, the authors applied simulation model. The discrete static berth allocation problem is introduced by Imai et al. (1997). The authors presented mathematical formulations for the single objective and multi-objective cases. Imai et al. (2001) considered dynamic berth allocation problem. They gave mathematical formulation and proposed lagrangian relaxation heuristic for the considered problem. Since 2001 many different versions of berth allocation problems have been considered in the literature, for example: berth allocation with service priorities (Imai et al. 2003), berth allocation at indented berths for mega-containerships (Imai et al. 2007).

Continuous berth allocation problem (BAPC) has been considered, for the first time, by Lim (1998). Author described the problem, proposed heuristic algorithm, and proof of its efficiency by using historical data from Port of Singapore Authority. Guan et al. (2002) presented heuristic algorithm (Heuristic H) for the continuous berth allocation problem. Guan and Cheung (2004)

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proposed two mathematical formulations and heuristic algorithm (Heuristic HB), improved version of Heuristic H. One heuristic algorithm for BAPC also has been developed in Imai et al. (2005b). Their heuristic is based on the solution of the dynamic berth allocation problem. Wang and Lim (2007) proposed a stochastic beam search algorithm and tested them on Singapore Port's test data. Two versions of Greedy Randomized Adaptive Search Procedure (GRASP) were developed in Lee et al. (2010). The results obtained by these approaches were compared with CPLEX and stochastic beam search.

Papers of Stahlbock and Voβ (2008) and Bierwirth and Meisel (2015) present broad reviews of the Berth Allocation Problem.

In this paper we consider Dynamic Berth Allocation Problem. The main research task in this problem is to find an assignment of ships to berths in the way to minimize the total time that all ships spend in the system (waiting time + service time). Word "Dynamic" assumes that when berths start to work it is not necessary that all ships are already in the port. Imai et al. (2001) proposed mix integer programming mathematical formulation for this problem. In this paper we start from their formulation and make, in the next step, improvements of their mathematical formulation.

The paper is organized in the following way. Imai et al. (2001) mathematical formulation is given in section 2. In section 3 we present improvements of the mathematical formulation. Numerical examples are also given in sections 2 and 3. Section 4 contains conclusions.

2. MATHEMATICAL FORMULATION OF THE DYNAMIC BERTH ALLOCATION PROBLEM

Imai et al. (2001) proposed the following mathematical formulation of the dynamic berth allocation problem:

$$Minimize \quad \sum_{i\in B}\sum_{j\in V}\sum_{k\in O}\left\{(T-k+1)C_{ij}+S_i-A_j\right\}x_{ijk}+\sum_{i\in B}\sum_{j\in W_i}\sum_{k\in O}(T-k+1)\,y_{ijk} \tag{1}$$

subject to:

$$\sum_{i\in B}\sum_{k\in O}x_{ijk}=1 \qquad \forall\, j\in V \tag{2}$$

$$\sum_{j\in V}x_{ijk}\le 1 \qquad \forall\, i\in B, k\in O \tag{3}$$

$$\sum_{l\in V}\sum_{m\in P_k}\left(C_{il}\;x_{ilm}+y_{ilm}\right)+y_{ijk}-\left(A_j-S_i\right)x_{ijk}\ge 0 \qquad \forall\, i\in B, j\in W_i, \quad k\in O \tag{4}$$

$$x_{ijk}\in\{0,1\} \qquad \forall\, i\in B, j\in V, k\in O \tag{5}$$

$$y_{ijk}\ge 0 \qquad \forall\, i\in B, j\in V, k\in O \tag{6}$$

where are (Imai et al. (2001)):

$i \quad (=1,\dots,I)\in B$	set of berths
$j \quad (=1,\dots,T)\in V$	set of ships
$k \quad (=1,\dots,T)\in O$	set of service order
S_i	time when berth *i* becomes idle for the berth allocation planning
A_j	arrival time of ship *j*
C_{ij}	handling time spent by ship *j* at the berth *i*
x_{ijk}	1 if ship j is serviced as the *k*th ship at berth *i* 0 otherwise

P_k	subset of O such that $P_k = \{p \mid p < k \in O\}$
W_i	subset of ships with $A_j \geq S_i$
y_{ijk}	idle time of berth i between the departure of the k-1th ship and arrival of the kth ship when ship j is serviced as the kth ship

Objective function (1), which represents the total time of all ships (waiting and service times), should be minimized. Constraint (2) guarantees that all ships will be served. Constraint (3) explains that only one ship can be served at the berth. Berth idle times are calculated according to Constraint (4). Constraints (5) and (6) define decision variables.

Corrigendum of the paper Imai et al. (2001) are made in the paper Imai et al. (2005a) where the authors made precise exploration about meaning of the decision variables x_{ijk} and y_{ijk}. They defined these decision variables in the following way (Imai et al., 2005a):

$x_{ijk} = 1$ if ship j is served as the $(T-k+1)$th last ship at berth, $x_{ijk} = 0$ otherwise

y_{ijk}: idle time of berth i between the departure of the $(T-k+2)$th last ship and the arrival of the $(T-k+1)$th last ship when ship j is served as the $(T-k+1)$th last ship.

Let us consider the following example (this example is based on instance 25x5-01 given in Cordeau et al. 2003). Suppose that we have 5 berths and 15 ships. Also, let us assume that the ship service times are given (Table 1), and:

- times when berths become idle for operations (S_i) are: 12, 12, 12, 12 and 12;
- ship arrival times (A_j) are: 71, 90, 39, 17, 12, 117, 94, 29, 43, 79, 2, 129, 123, 43 and 5.

Table 1. Ship service time at each berth

	Ship														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Berth 1	20	44	22	14	12	30	28	6	26	22	20	16	26	14	18
Berth 2	20	44	22	14	12	30	28	6	26	22	20	16	26	14	18
Berth 3	40	88	44	28	24	60	56	12	52	44	40	32	52	28	36
Berth 4	40	88	44	28	24	60	56	12	52	44	40	32	52	28	36
Berth 5	40	88	44	28	24	60	56	12	52	44	40	32	52	28	36

By solving the mix integer program, generated according to the presented data and the mathematical formulation (1)-(6), we obtained the following solution: $x_{1,3,14} = 1$, $x_{1,5,12} = 1$, $x_{1,6,15} = 1$, $x_{1,11,13} = 1$, $x_{2,1,13} = 1$, $x_{2,2,15} = 1$, $x_{2,7,14} = 1$, $x_{2,8,11} = 1$, $x_{2,15,12} = 1$, $x_{3,12,14} = 1$, $x_{3,13,15} = 1$, $x_{4,4,14} = 1$, $x_{4,9,15} = 1$, $x_{5,10,15} = 1$, $x_{5,14,14} = 1$, $y_{1,11,1} = 51$, $y_{2,11,1} = 38$, $y_{3,11,1} = 117$, $y_{4,11,1} = 5$, $y_{5,11,1} = 39$. All other variables have value zero. The objective function value of this solution is equal to 55. Taking into consideration x_{ijk} variables we can see that:

- at the berth one will be served ships: 5, 11, 3 and 6;
- at the berth two will be served ships: 8, 15, 1, 7 and 2;
- at the berth three will be served ships: 12 and 13;
- at the berth four will be served ships: 4 and 9;
- at the berth five will be served ships: 14 and 10.

But if we carefully take a look on variables y_{ijk} we can notice that all variables ($y_{1,11,1}$, $y_{2,11,1}$, $y_{3,11,1}$, $y_{4,11,1}$, and $y_{5,11,1}$) refer to ship 11. That means that ship 11 should wait service at all berths. Obviously, the obtained solution is problematic and should be carefully examined.

Let us investigate the input data. We can notice that ship 11 arrives before time point when berths start to work. Also, we can notice that objective function value, 55, does not include the obtained values for y_{ijk}. Taking into consideration that ship 11 arrive before berths start to work, it is obvious that $11 \notin W_i, i = 1, 2, 3, 4, 5$. As a consequence, any y_{ijk} where j = 11 is not included into objective function. Because $l \in V$ (the set V contains all the ships, including ship 11) in the first part of (4), $\sum_{l \in V} \sum_{m \in P_k} \left(C_{il}\ x_{ilm} + y_{ilm}\right)$, decision variables $y_{i,11,k}$ are included into these constraints. Large enough values for $y_{i,11,k}$ caused that these constraints are not broken even though obtained solution is unfeasible.

We make in this paper the suggestions how to improve mathematical formulation. The suggestions are proposed in the next section. The proposed suggestions are closely related to the above observations.

2. MODIFICATION OF THE MATHEMATICAL FORMULATION

There are two possible modifications of the mathematical formulation proposed in (Imai et al. 2001). The mathematical formulation could be improved in the following two ways: (a) modification of the objective function; (b) modification of the constraints.

2.1 Modification of the objective function

As we already noticed, the decision variables y_{ijk} , for ships j that arrive before berths start to work ($j \notin W_i$), are not included in the objective function (1). In order to resolve the problem in the mathematical formulation (1)-(6) we modify the objective function. We use the set V instead of W_i in the last part of the objective function in, i.e.: $\sum_{i \in B} \sum_{j \in V} \sum_{k \in O} \left(T - k + 1\right) y_{ijk}$. The other parts of the mathematical formulation remain unchanged. The new mathematical formulation reads:

$$Minimize \quad \sum_{i \in B} \sum_{j \in V} \sum_{k \in O} \left\{\left(T - k + 1\right) C_{ij} + S_i - A_j\right\} x_{ijk} + \sum_{i \in B} \sum_{j \in V} \sum_{k \in O} \left(T - k + 1\right) y_{ijk} \tag{7}$$

subject to

$$\sum_{i \in B} \sum_{k \in O} x_{ijk} = 1 \qquad \forall\, j \in V \tag{8}$$

$$\sum_{j \in V} x_{ijk} \le 1 \qquad \forall\, i \in B, k \in O \tag{9}$$

$$\sum_{l \in V} \sum_{m \in P_k} \left(C_{il}\ x_{ilm} + y_{ilm}\right) + y_{ijk} - \left(A_j - S_i\right) x_{ijk} \ge 0 \qquad \forall\, i \in B, j \in W_i, \quad k \in O \tag{10}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall\, i \in B, j \in V, k \in O \tag{11}$$

$$y_{ijk} \ge 0 \qquad \forall\, i \in B, j \in V, k \in O \tag{12}$$

The meanings of the objective function and the constraints are the same as in formulation (1)-(6).

2.2 Modification of the constraints

The second idea for resolving problem in mathematical formulation (1)-(6) is to use variables y_{ijk} only for ships that arrive after the time point when berths start to work $(y_{ijk} \geq 0 \quad \forall\, i \in B,\, j \in W_i,\, k \in O)$. (There is no reason to delay handling of any ship already in the port, scheduled to be served as the next one at the berth). We must rewrite the first part of the constraint (10). Instead of the $\sum_{l \in V} \sum_{m \in P_k} \left(C_{il}\, x_{ilm} + y_{ilm}\right)$, we write $\sum_{l \in V} \sum_{m \in P_k} C_{il}\, x_{ilm} + \sum_{l \in W_i} \sum_{m \in P_k} y_{ilm}$. In this way, the constraint takes into account service times of all ships, and the berth idle times for the ships that arrive after berths start to work. The new mathematical formulation reads:

$$Minimize \quad \sum_{i \in B} \sum_{j \in V} \sum_{k \in O} \left\{(T-k+1)\, C_{ij} + S_i - A_j\right\} x_{ijk} + \sum_{i \in B} \sum_{j \in W_i} \sum_{k \in O} (T-k+1)\, y_{ijk} \tag{13}$$

subject to

$$\sum_{i \in B} \sum_{k \in O} x_{ijk} = 1 \qquad \forall\, j \in V \tag{14}$$

$$\sum_{j \in V} x_{ijk} \leq 1 \qquad \forall\, i \in B, k \in O \tag{15}$$

$$\sum_{l \in V} \sum_{m \in P_k} C_{il}\, x_{ilm} + \sum_{l \in W_i} \sum_{m \in P_k} y_{ilm} + y_{ijk} - \left(A_j - S_i\right) x_{ijk} \geq 0 \qquad \forall\, i \in B, j \in W_i, \quad k \in O \tag{16}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall\, i \in B,\, j \in V, k \in O \tag{17}$$

$$y_{ijk} \geq 0 \qquad \forall\, i \in B,\, j \in W_i, k \in O \tag{18}$$

The meanings of the objective function and the constraints are the same as in formulation (1)-(6).

We have solved the example given in the section 3 as the mix-integer program obtained according to the mathematical formulation (13)-(18). The obtained solution is the following: $x_{1,1,13} = x_{1,2,14} = x_{1,6,15} = x_{1,8,11} = x_{1,9,12} = x_{1,15,10} = x_{2,3,12} = x_{2,7,14} = x_{2,10,13} = x_{2,11,11} = x_{2,13,15} = x_{3,4,15} = x_{5,5,13} = x_{5,12,15} = x_{5,14,14} = 1$, $y_{1,1,13} = 2$, $y_{1,9,12} = 7$, $y_{2,3,12} = 7$, $y_{2,10,13} = 18$, $y_{3,4,15} = 5$, $y_{5,12,15} = 58$ and $y_{5,14,14} = 7$. The objective function value of this solution equals 424. The solution represents the following allocation ships to berths:

- berth 1 serves the ships: 15, 8, 9, 1, 2 and 6;
- berth 2 serves the ships: 11, 3, 10, 7 and 13;
- berth 3 serves the ship 4;
- berth 4 does not serve any ship;
- berth 5 serves the ships: 5, 14 and 12.

3. CONCLUSION

The Berth Allocation Problem has been extensively studied in the literature. We considered in this paper dynamic discrete berth allocation problem. The widely accepted mathematical formulation of this problem is proposed by Imai et al. (2001). We noticed some weaknesses in the formulation of Imai et al. (2001). In order to mitigate these weaknesses, we proposed two ways for mathematical formulation improvement. The first way represents the modification of the proposed objective function. Our second proposal is related to the modification of constraints. We showed that the new mathematical formulation of the discrete berth allocation problem is based on the modification of constraints in the formulation proposed by Imai et al. (2001).

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