CONTRACTS FOR SUPPLY CHAIN COORDINATION WITH PRICE-DEPENDENT STOCHASTIC DEMAND

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Abstract: *Supply chain is a decentralized system where material, financial, information and decision flows connect members. When one or more members of the supply chain try to optimize their own profits, the total system may be inefficient. Supply chain contracts are used to provide some incentives to adjust the relationship of supply chain partners to coordinate the supply chain, i.e., the total profit of the decentralized supply chain is equal to that achieved under a centralized system. Double marginalization is a well-known cause of supply chain inefficiency and the problem occurs whenever the supply chain's profits are divided among more members and at least one of the members influences price-dependent demand. When the demand is stochastic than the newsvendor model can be applied. In a standard newsvendor problem the price is assumed to be fixed. The aim of this paper is to analyze contracts for the combined problem of supply chain coordination with price-dependent stochastic demand.*

Keywords*: supply chain, coordination, contracts, price-dependent stochastic demand*

1. INTRODUCTION

Supply chain is a decentralized system where material, financial, information and decision flows connect members. Recent years have seen a growing interest among researchers and practitioners in the field of supply chain management. Supply chain management is about matching supply and demand with inventory management. When one or more members of the supply chain try to optimize their own profits, system performance may be hurt (Fiala, 2005). Among the solutions, supply chain contracts, which have drawn much attention from the researchers recently (for review Cachon, 2003, Tsay et al., 1999), are used to provide some incentives to adjust the relationship of supply chain partners to coordinate the supply chain, i.e., the total profit of the decentralized supply chain is equal to that achieved under a centralized system. The format of supply chain contracts varies in and across industries.

Double marginalization (Spengler, 1950) is a well-known cause of supply chain inefficiency and the problem occurs whenever the supply chain's profits are divided among two or more members and at least one of the members influences price-dependent demand. Each firm only considers its own profit margin and does not consider the supply chain's margin. When the demand is stochastic than the newsvendor model can be applied. The newsvendor model is not

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complex, but it is sufficiently rich to study important questions in supply chain coordination. In a standard newsvendor problem the price is assumed to be fixed.

The aim of this paper is to analyze contracts for the combined problem of supply chain coordination with price-dependent stochastic demand. Contracts are evaluated by desirable features:

- coordination of the supply chain,
- flexibility to allow any division of the supply chain's profit,
- easy to use.

2. PRICE-DEPENDENT DETERMINISTIC DEMAND

Double marginalization (Spengler, 1950) is a well-known cause of supply chain inefficiency. Double marginalization problem occurs whenever the supply chain's profits are divided among two or more firms and at least one of the firms influences demand. Each firm only considers its own profit margin and does not consider the supply chain's margin.

We consider a supply chain with a supplier and a retailer that sells a product. The supplier produces each unit for a cost c and sells each unit to the retailer for a wholesale price w. The retailer chooses an order quantity q and sells q units at price $p(q)$, assuming that $p(q)$ is decreasing, concave and twice differentiable function.

Centralized solution assumes a single agent has complete information and controls the entire supply chain (this is referred as the firs-best solution) to maximize supply chain profit

$$z(q) = (p(q) - c)\, q.$$

Since profit is strictly concave in quantity, the optimal quantity q^0 satisfies

$$\frac{dz(q)}{dq} = 0\,.$$

Decentralized solution assumes the firms have incomplete information and make choices with the objective of maximizing their own profits. The retailer's profit and the supplier's profit are

$$z_r(q) = (p(q) - w)\, q \quad , \quad z_s(q) = (w - c)\, q.$$

Optimal solution of the problem we denote q^*.

If the centralized and decentralized solutions differ, investigate how to modify the firm's payoffs so that new decentralized solution corresponds to the centralized solution.

It can be shown that the retailer orders less than the supply chain optimal quantity ($q^0 > q^*$) whenever the supplier earns a positive profit and it holds

$$z(q^0) > z_r(q^*) + z_s(q^*)$$

Marginal cost pricing ($w = c$) is one solution to double marginalization problem, but the supplier earns a zero profit. A better solution is a profit sharing contract, where the supplier earns $\lambda\, z(q)$ and the retailer earns $(1\text{-}\lambda)\, z(q)$, for $0 \le \lambda \le 1$. The wholesale price w is now irrelevant to each firm's profits and the supply chain earns the optimal profit.

3. STOCHASTIC DEMAND

There are some concepts for contracts with stochastic demand (Anupindi and Bassok, 1999, Lariviere, 1999). We consider a supply chain in one-period setting in which a supplier sells to a retailer facing stochastic demand from consumers. We assume that stochastic demand x has a

continuous distribution $F(x)$ that is invertible. The demand distribution and cost information are common knowledge. Define the failure rate function of the x distribution as

$$g(x) = \frac{f(x)}{1 - F(x)}$$

and the generalized failure rate function as

$$h(x) = xr(x).$$

Assume the demand distribution has strictly increasing generalized failure rate property (IGFR). Many distributions have the IGFR property, including the normal, the exponential, the gamma, and the Weibull.

We define the following quantities:

q retailer's total order quantity;
c supplier's production cost;
p retail price.

The setting can be characterized as a newsvendor problem.

Centralized solution

Centralized solution is a benchmark for the decentralized supply chain. The centralized chain is considered as an integrated firm that controls manufacturing and sales to consumers. The profit of an integrated firm for stocking level q is

$$z(q) = (p - c)q - p\int_0^q F(x)dx .$$

The problem is concave in q and the optimal solution is given by

$$q^0 = F^{-1}\left(\frac{p - c}{p}\right).$$

The maximum system profit $z(q^0)$ is completely determined by the stocking level q^0.
Decentralized solution can be improved by contracting. The contract coordinates the chain if it induces the choice of the centralized system's optimal stocking level q^0.

Wholesale price contracts

With a wholesale price contract the supplier charges the retailer w per unit purchased. The retailer faces a problem analogous to that of the integrated chain with. The principal difference is that the retailer must buy stock at the wholesale price w instead of producing it at cost c.

The retailer's profit is

$$z_R(q) = (p - w)q - p\int_0^q F(x)dx .$$

The retailer's problem is concave in q and the optimal solution is given by

$$q(w) = F^{-1}\left(\frac{p - w}{p}\right).$$

The supplier acts as a Stackelberg leader and anticipates how the retailer will order for any wholesale price. The supplier anticipates a demand curve $q(w)$ and the profit

$$z_S(w) = (w - c)q(w) = (w - c)F^{-1}\left(\frac{p - w}{p}\right).$$

The supplier knows exactly what retailer will order at every wholesale price and bears no responsibility for the product. All uncertainty regarding supply profits is foisted onto the retailer. The wholesale price contract coordinates the chain only if the supplier earns a non-positive profit. So the supplier clearly prefers a higher wholesale price. As a result, the wholesale price contract is generally not considered a coordinating contract. The richer contracts differ from wholesale price contracts by allowing the supplier to assume some of the risk arising from stochastic demand. As an example we introduce buy back contracts.

Buy back contracts

With a buy back contract (Pasternack, 1985) the supplier charges the retailer w per unit purchased, but pays the retailer b per unit remaining at the end of the season. A retailer should not profit from left over inventory, so assume $b \le w$. There is assumed that a returns policy on the decentralized chain introduces no additional cost beyond that incurred by the centralized system.

The retailer's profit is

$$z_R(q) = (p - w)q - (p - b)\int_0^q F(x)dx .$$

The retailer still faces a newsvendor problem. The optimal solution is

$$q(w,b) = F^{-1}\left(\frac{p - w}{p - b}\right).$$

4. PRICE-DEPENDENT STOCHASTIC DEMAND

Little work has been done on the combined problem of supply chain coordination with price-dependent stochastic demand. The contracts proposed for coordination with price-independent stochastic demand are not applicable for coordination of supply chains with price-dependent stochastic demand.

We will analyze the multiplicative form of price-dependent stochastic demand

$$D(p, x) = y(p)x ,$$

a function of p and x, where x is a random variable independent of p and $y(p)$ is continuous, nonnegative, twice differentiable function. The expectation of D is specified by a function $y(p)$ for any given price p:

$$E[D(p, x)] = y(p).$$

The flows in the supplier-retailer supply chain with stochastic price-dependent demand are captured in Fig. 1. Material and unit financial flows are represented by continuous and dash lines, respectively.

The expected profit for centralized solution for any output level q and price p is:

$$z(p, q) = E\{p[\min(q, D(p, x)] - cq\} = E\{(p - c)q - p\max(0; q - D(p, x))\} =$$

$$= (p - c)q - py(p)\int_0^{\frac{q}{y(p)}} F(x)dx .$$

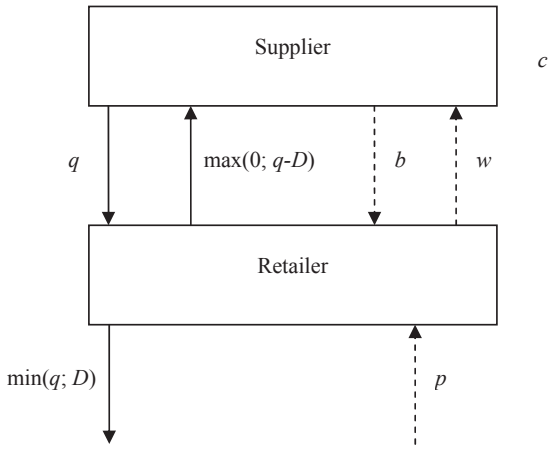

Figure 1. Supplier-retailer supply chain with stochastic price-dependent demand

The objective is to choose (p^0, q^0) to maximize the expected profit $z(p, q)$.

By fixing price p the problem reduces to standard newsvendor problem without pricing and the optimal level of inventory

$$q^0 = y(p)F^{-1}\left(\frac{p-c}{p}\right).$$

By substituting it into the expected profit

$$z(p) = y(p)[(p-c)F^{-1}\left(\frac{p-c}{p}\right) - p \int_0^{F^{-1}\left(\frac{p-c}{p}\right)} F(x)dx].$$

The problem is now with only one decision variable p and the optimal price p^0 can be obtained by solving

$$\frac{dz(p)}{dp} = 0.$$

The assumptions of the existence and uniqueness of the optimal solution (p^0, q^0) are concavity of deterministic part of demand function $y(p)$ and IGFR property of stochastic part o demand function x.

The proposed contract for coordination of the decentralized supply chain is a hybrid of wholesale price and buy-buck contract. The wholesale price w and the buy-buck price b are specified:

$$w = \lambda\,(p - c) + c,$$

$$b = \lambda\, p, \text{ where } 0 \le \lambda \le 1.$$

By the setting of the prices w and b the retailer's profit and the supplier's profit for any chosen output level q and price p are

$z_R = E\,\{p[\min(q, D(p, x)] - wq + b\,\max(0; q\text{-}D(p, x))\} = E\,\{\,(p - w - c)q - (p - b)\,\max(0; q\text{-}D(p, x))\,\} =$

$= (1 - \lambda)\,E\{(p - c)q - p\,\max(0; q\text{-}D(p, x))\,\} = (1 - \lambda)\,z\,,$

$z_S = E\{(w - c)q + b\,\max(0; q\text{-}D(p, x))\} = E\{\lambda\,(p - c)q - \lambda\,p\,\max(0; q\text{-}D(p, x))\,\} = \lambda\,z.$

From previous expressions of the retailer's profit and the supplier's profit, it is clear that the retailer and the supplier solve the same problem as the centralized supply chain and the sum of the retailer's profit and the supplier's profit is equal to the profit of the centralized supply chain. The parameter λ characterizes a splitting of the total profit between the retailer and the supplier.

5. CONCLUSION

There is a vast literature on supply chain contracts recently. However, little work has been done on the relationships of those supply chain contract models and on the combined problem of supply chain coordination with price-dependent stochastic demand. The proposed contract for supply chain coordination with price-dependent stochastic demand has desirable features. The supply chain is fully coordinated, i.e., the total profit of the decentralized supply chain is equal to that achieved under a centralized system. Flexibility to allow any division of the supply chain's profit is managed by the selected parameter λ in the setting of the wholesale price w and the buy back price b. It has relative advantages in implementation. The supplier needs to monitor the price only, not the quantity sold. The analysis of the simple cases of contracts gives recommendations for more complex real problems.

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