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APPLICATION OF QUEUING THEORY AND SIMULATION IN DIMENSIONING SUBSYSTEMS OF A LOGISTIC CENTER

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Abstract: *Insufficiently dimensioned technological elements of a certain subsystem in a logistic center can make impossible not only normal functioning of the subsystem, but also of the whole logistic system in which the subsystem exists. On the other hand, over dimensioning of technological elements can lead to unnecessary expenses. The subject of this paper is an application of queuing theory and discrete event simulation in dimensioning subsystems of logistic center, which is one of the key parts in the process of designing of logistic center.*

Keywords*: logistic center, technological element, dimensioning, queuing theory, simulation*

1. INTRODUCTION

Within a logistic center there can be a large number of subsystems that can be considered as separate systems: warehouse systems, container terminal, cross-docking terminal, customs terminal, systems for the production, processing and assembly, system for fuel replenishment, system for care, maintenance and repair of vehicles, etc (Zečević 2006). In each of these subsystems appear certain technological demands (Vukićević, 1995). For example, in the warehouse systems, those demands may be: waiting for loading or unloading, loading or unloading, storage, filling order picking zone from the reserve zone, order picking, packing, measuring, etc., in the system for fuel replenishment: fuel storage, waiting for refueling, refueling, etc. Technological demands require appropriate technological elements (Vukićević, 1995). In the warehouse system we need following elements: parking lot, loading dock, forklift, aisles, conveyor, pallet rack, storage zone, order picking zone, forklift driver, order picking personnel, etc., in the case of system for fuel replenishment: underground fuel storage tanks, a place to wait for a refueling, place for refueling, etc. Given this division of logistic center, the process of dimensioning in subsystems of logistic center is aimed at determining the required number of technological elements, their dimensions, capacities, etc. Dimensioning in logistic centers has direct implications on expenses, such as construction, material handling, inventory holding and replenishment; there can also be some lost profits because of time loss while the vehicle is waiting in the logistic center or due to an impossibility to implement services, due to an inadequate implementation of services, etc. That is the reason why the process of dimensioning is realized with a significant amount of attention, inventiveness and creative spirit, and it often goes with an application of branches from mathematics and operational researches.

For each of these subsystems there is a large number of papers that talk about dimensioning of technological elements. Gu et al. (2007) present a review of references that are related to

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determining of required warehouse capacity. In his paper, Gu (2005) gives a solution to a problem of dimensioning of storage and order picking zone. Apart from analytical methods, the best support in resolving dimensioning problems is a simulation. Carteni and Luca (2012) give a detailed review of simulation modeling of processes in ports, container and other terminals in order to achieve better dimensioning, better usage of resources and acceleration of the process.

The following material presents the queuing theory, the analogy of queuing system with processes in logistic center and possibilities of application in dimensioning. In the third section, by applying the queuing theory and simulation, is solved the real problem of determining a required number of technological elements in warehouse system and system for fuel replenishment within a logistic center for mineral fertilizers that is supposed to be designed. The fourth section contains the final considerations, the review of more complex examples of dimensioning than those represented in this paper, and the overview of dimensioning processes on a larger scale.

2. QUEUING THEORY IN LOGISTIC SYSTEMS

Queuing theory represents important branch of operations research, which researches connection between flow of demands for service and the ability of executing demands in queuing system. Within the logistic systems there are many processes that can be presented as queuing system. Elements of the queuing system can have the following meanings in logistic centers, warehouse systems and material handling:

- servers – technological elements: forklifts, cranes, receiving and shipping docks, order picking personnel, gas pumps, machines, etc.
- service – technological demands: loading, unloading, transport, order picking, refueling, production, packing, etc.
- customers: means of transport all modes of transport, containers, pallets, boxes, bags, orders, items, etc.
- queue: trucks on parking lot, orders in the information system which are waiting for an order picking, items on buffer, etc.

Symbols, which were formed by English mathematician David G. Kendall (1953), are standard used for describing queuing models. In this paper, queuing models are described by shorten Kendall's notation $I/II/n/m$. The first letter specifies the inter-arrival time distribution. The second letter specifies the distribution function of the service times. For example, Poisson process – exponentially distributed random variables are notated by M (meaning Markovian or memoryless), for a general distribution the letter G is used, D for deterministic times, etc. The third and fourth letters specify the number of servers and the number of places in queue. Basic input performance measures of queuing systems are: n – number of servers, m – number of places in queue, λ – arrival rate and μ – service rate of one server. By applying certain mathematical relations in queuing models we establish basic output performance measures of queuing systems: p_k – probability of having k customers at service, p_{n+r} – probability of having r customers in queue, $\rho=\lambda/\mu$ – traffic intensity, $\alpha=\lambda/n\mu$ – server utilization, L_q – average number of customers in queue, L_s – average number of customers at service, L– average number of customers in system, W_q – average time spent in queue, W_s – average time spent at service, W – average time spent in system, etc. On the other hand, dimensioning by applying queuing theory usually means defining input performance measures of queuing systems, like required number of servers n and number of places in queue m. Sometimes it involves defining arrival intensity which queuing system can service λ or needed service rate of one server μ.

A larger application of the queuing theory is impossible due to the fact that almost all models, for which exact solutions exist, refer only to the steady state and to the models where the arrivals are according to Poisson process, and where the service time is in line with an exponential distribution. When the arrivals and/or service time are described by some other probability distribution and/or in a case of non-stationarity, a simulation is used. However, even in those cases, the queuing theory is used as a good verification tool of simulation models. On the other hand, Vukićević (1995) says that throughput capacity and other service characteristics depend relatively little on probability distribution of service time; in most cases, they depend on its average value, and therefore it is often neglected that the service time does not correspond with an exponential distribution, but with some other distribution. Nevertheless, using of exponential distribution of service time can be the cause to an over dimensioning of technological elements (Vukićević, 1995). This happens because of attributing greater stochasticity to the time needed for service than what it really is.

3. OPTIMIZATION OF THE NUMBER OF TECHNOLOGICAL ELEMENTS NEEDED

In this paper, the subsystems for storage and fuel replenishment within a logistic center for mineral fertilizers are dimensioned. The aim of this paper is determining of the following: the number of loading docks, parking lots and forklifts within a warehouse system, as well as the number of replenishment places and places for waiting to be replenished in these subsystems. The first to be presented is input data related to both subsystems, and then the solution to problems in dimensioning in warehouse system, whereas we only give the final result for number of technological elements in the system for fuel replenishment and some aside acknowledgments because of obvious analogy with the situation in warehouse system.

At the warehouse system trucks will arrive according to a Poisson process (i.e. exponential inter-arrival times) during the whole day. The arrival rate λ is 11 trucks/hour. In the 45% of cases, those are trucks in receiving, in which are 16 to 24 pallets with one kind of mineral fertilizer, while the rest of cases are trucks in shipping. In those trucks 14 to 19 pallets can appear, but with different kinds of mineral fertilizers. Since we do not know probability distribution of number of pallets in trucks, we suppose discrete uniform distribution in both cases. If there is some free space, truck goes directly to the loading zone, otherwise the truck is waiting in the parking lot. All material handling operations in warehouse are performed by forklifts that take pallets and put them directly from the truck to the storage and later from the storage directly to the truck. In the warehouse, there is not order picking of bags with fertilizer from pallet, but whole pallets are shipped the way they were. Upon arrival, the homogeneous group of pallets is put in the block stacking storage system according to frequency of a sort of mineral fertilizer – the most frequent sort is assigned to the location that is the nearest to the loading docks. By taking into account the frequency of mineral fertilizers and the position of dimensioned storage zones, we came to a conclusion that the working cycle of forklifts (T_c) can be described by 0,6 + *EXPO*(0,5) in minutes. Logically, the working cycle of forklifts for loading the pallets from storage to truck will be described the same way, but the events will be differently scattered in time – the random function will be different. After having done loading/unloading, about 30% of trucks go to the system for fuel replenishment, while others are leaving the logistic center. However, besides these trucks, this system is used by external trucks, too. The arrival of those trucks is according to Poisson process with an arrival rate of 4 trucks/hour. The time required for fuel replenishment may be described by *TRIA*(3,5,7) in minutes. The costs of construction of one loading dock is 5000 $ (c_1), and the cost of a parking lot is 2000 $ (c_4). The price of one forklift is 14000 $ (c_2). Battery charging of a forklift is done the following way – the battery is taken away from the forklift, and the new one is put into it so that it can continue to work. In the case of dimensioning of the fuel replenishment system, the costs of construction of replenishment places and places for waiting to be replenished are 3000 $ and 2000 $, respectively. The trucks are at a loss while waiting in logistic center because they make money while on route where they could have earned 5$/hour ($c_5$). The project lifetime – the period of exploitation of a logistic center is 10 years (τ). The warehouse is supposed to be open 7 days a week, and the working time of the warehouse is 16 hours, in two shifts. A forklift driver is paid 300$/month ($c_3$).

The number of forklifts, loading docks and parking lots can be determined by taking into account of all the costs that are in the function of number of servers – loading docks (forklifts):

$$C(n) = (c_1 + c_2 + 2c_3\tau)n + c_4 m + c_5 W_q \tau\lambda$$

(1)

where:

- n – number of forklifts (number of loading docks)[1];
- m – number of parking lots;
- W_q – average time spent on parking lot (h);
- c_1 – costs of construction of one loading dock ($);
- c_2 – price of one forklift ($);
- c_3 – salary of one forklift driver ($/h);
- c_4 – costs of construction of a parking lot ($);
- c_5 – costs of waiting for loading ($/h);
- τ –project lifetime – exploitation period of logistic center (h);
- λ – arrival rate of the trucks in the warehouse system (truck/h).

The task is to find n and m that depends on it, so that the equation (1) has a minimum value. The number of parking lots can be determined so that, during a certain percent of time, the system can receive in adequate way (on loading dock or parking lot) every truck that arrives. Let's suppose that it is 99% of time. Therefore, the task is to find the first m for which is true:

$$\sum_{k=0}^{n} p_k + \sum_{r=1}^{m} p_{n+r} \geq 0{,}99 \qquad (2)$$

The real system corresponds to the model with n servers, partially mutual help and infinite number of places in queue $M/M/n^{(l)}/\infty$, where the parameter l tells the maximal number of forklifts that can load/unload a truck, so that the productivity and safety of the operations do not degrade. The requirement for the number of places in queue to be unlimited is almost always present during the material handling processes because technological demands, in most cases, cannot be cancelled – they have to be fulfilled sooner or later, and the space for waiting has to be found, even if it is sometimes inadequate. Formulas for steady-state probabilities and average time spent in queue for this model are given in the table 1. For the purpose of understanding the model, the flow diagram is represented in the figure 1.

Table 1. Formulas for steady-state probabilities and average time spent in queue for $M/M/n^{(l)}/\infty$

$M(\lambda)/M(\mu)/n^{(l)}/\infty$
$p_0 = \dfrac{1}{\sum_{k=0}^{h}\left(\frac{\rho}{l}\right)^k \frac{1}{k!} + \left(\frac{\rho}{l}\right)^h \frac{1}{h!}\left(\frac{\alpha^{n-h+1}}{1-\alpha} + \sum_{k=1}^{n-h}\alpha^k\right)} \quad for\ \alpha < 1$
$p_k = \begin{cases} \left(\frac{\rho}{l}\right)^k \frac{p_0}{k!}, & k = \overline{0,h} \\ \alpha^{k-h}\left(\frac{\rho}{l}\right)^h \frac{p_0}{h!}, & k = \overline{h+1,n} \end{cases} \qquad h = \left[\frac{n}{l}\right]$
$p_{n+r} = \left(\frac{\rho}{n}\right)^r \frac{\rho^n}{n!} p_0, \qquad r = 1,2,3,\ldots$
$W_q = \dfrac{\alpha^{n-h+1}\left(\frac{\rho}{l}\right)^h \frac{p_0}{h!}}{(1-\alpha)^2\lambda}$

[1] Loading dock and forklift are considered to be the only server because the time spent on the loading dock is determined only by the load/unload time. If some other operations were included, like, for example, the preparation of the truck for loading/unloading, that would be different.

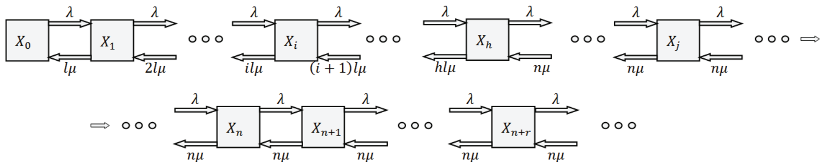

Figure 1. Flow diagram for the model $M(\lambda)/M(\mu)/n^{(l)}/\infty$

By taking into account the ratio of number of trucks in receiving and shipping and the probability distributions of number of pallets in trucks, we get an average number of 18,075 pallets per truck. Given that the working cycle of a forklift is 1,1 min in average, the average time for one forklift to load/unload a truck is 19,88 min, which implies that an intensity rate of one forklift μ is cca 3 trucks/hour. It is estimated that the optimal l is actually 2, so that the model can be described as follows: $M(\lambda=11)/M(\mu=3)/n^{(2)}/\infty$. The arrival of the truck is in line with Poisson process but the time spent on the loading dock is not, it is rather the same general distribution, that has to be included by the simulation. Figure 2 represents a part of a simulation model created in software ARENA, that also modules the fuel replenishment system. The part of simulation model that refers to the warehouse system can be represented this way: $M(\lambda=11)/G(\mu=3)/n^{(2)}/\infty$, and the part of the system related to the fuel replenishment system: $G(\lambda=7,3)/TRIA(3,5,7)/n/\infty$. In this simulation, 365 replications of 16 hours (one working day in a logistic center) are run.

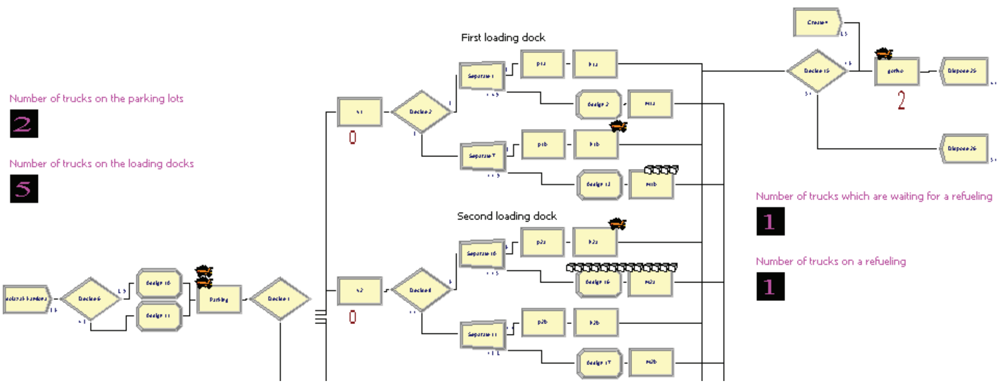

Figure 2. Simulation model

In order for the warehouse system to function in steady state, the service intensity of all servers $n\mu$ has to be higher than the arrival rate λ, which means that n has to be higher than 3.67. This relation shows that the warehouse system must have more than 3 forklifts in order to avoid a large number of trucks. This requirement does not have to be fulfilled in a case where the duration of the simulation is limited, but it is quite unlikely for optimal n to be less than 4. The table 2 presents the number of parking lots and average time spent on the parking lot in the function of number of forklifts, as a result of simulation and analytically, by queuing theory. The expenses chart are given in the figure 3. It is evident that the optimal number of forklifts and places on loading dock is 5 in both types of solution: by a simulation and analytically.

Table 2. m and W_q depend on n for models $M/G/n^{(2)}/\infty$ and $M/M/n^{(2)}/\infty$

	$M/G/n^{(2)}/\infty$		$M/M/n^{(2)}/\infty$	
n	m	W_q (h)	m	W_q (h)
4	14	0,3096	48	0,7061
5	5	0,0375	9	0,0530
6	0	0,0002	3	0,0074
7	0	0,0000	0	0,0010

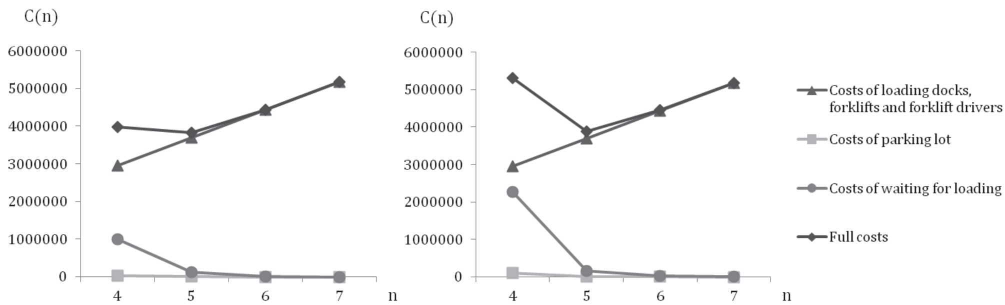

Figure 3. Costs depending on *n* for models $M(\lambda=11)/G(\mu=3)/n^{(2)}/\infty$ and $M(\lambda=11)/M(\mu=3)/n^{(2)}/\infty$

For this number of forklifts – loading dock, 5 parking places are required. It is noted that this number is over dimensioned by applying an analytical model because the result is 9. Concerning the subsystem for fuel replenishment, only 1 place for refueling and 2 places for waiting to be replenished are required.

4. CONCLUSION

Obtained results justify the affirmations about the possibility of over dimensioning by applying the queuing theory, but nevertheless, this theory can be used as a verification tool for even more complex simulation models than this one. In the presented example, the input data are given independently of the fact that commodity and transport flows are type of flows that cannot be predicted with accuracy. Therefore it is always needed to represent different scenarios by varying the input data. In some dimensioning problems, there can be a non-stationarity during an exploitation period (phasing), a year (seasonal character) or a day. The non-stationarity sometimes might impose including of a random function instead of random variables in the simulation model. Thereafter, the dimensioning is inseparable from the technology of realization and layout, and it is supposed to be done simultaneously for the whole subsystem of the logistic center. The more complex a subsystem is, the larger is the number of mutual relations that are to be respected when dimensioning, and consequently, the more difficult is a task of an engineer, who has to cope with all those challenges.

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