FINDING LOCATIONS OF DISTRIBUTION CENTRES WITH TIME WINDOW RESTRICTED CUSTOMER REQUESTS

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***Abstract**: Designing an efficient distribution network is one of the main tasks of supply management today. We consider a case when distribution network contains distribution centres and goods need to be delivered via vehicles to customers with time window constraints. The main goal in the considered problem is determining good locations for distribution centres, together with minimizing their number, the number of vehicles and the total driving time.*

Keywords: *cross-docking, location routing problem, time-windows.*

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1. INTRODUCTION

One of the major tasks of supply chain management today is an efficient design of distribution networks. In particular, determining good locations for distribution centres via simulations that take customer geographic locations, road network, time and capacity constraints into account is always challenging. An efficient supply chain thus involves high level of synchronization between facility location and vehicle routing decisions. The facility location problem as a strategic problem is often influenced with social, political and economical conditions, consequently reducing the number of potential facility locations to only few locations. On the other hand, vehicle routing as a tactical problem is a subject of daily adjustment to customer's requests.

The necessity of an integrated approach to facility location and vehicle routing problems indicates the paper of Maranzana [5]. A comprehensive research of literature and problem variations of combined location routing problem can be found in the papers of Min Min, Jayaraman and Srivastava [6] and Nagy, Salhi [8]. The papers of Prins and co-authors [9],[10] deal with capacitated location routing problem. As one of the future research directions, the importance of time component, as the essence of the Just-In-Time logistics principle is emphasised in [6]. Only few papers deal with this aspect. In [8] authors argue that "*time windows relate to a much smaller time horizon than facility location, this horizon mismatch may have deterred researchers from including this aspect*". Semet and Taillard in [11] considered special case of road-train routing problem. Mirzaei and Krishnan in [7] consider location routing problem with time dependent travel times and time windows. A mathematical formulation of the problem is presented and verified on examples with only few customers. In the paper of Jacobsen and Madsen [4], a two level location routing problem of realistic dimensions was considered. Similarly, in [1],[3] deals with positioning of cross-docking points in newspaper delivery problem.

In this paper we focus on the problem of locating the distribution centres respecting the customer requests and time windows. Therefore, the problem can be classified as a location-routing problem with time windows. Our aim is to solve the large scale problems up to few thousands customers. We define algorithm that simultaneously chooses depot locations and optimizes vehicle routes. The quality of the algorithm is verified on instances with 500-2000 customers.

The rest of the paper is organized as follows: In Section 2 we give the problem description, the proposed solution is presented in Section 3 and the results in Section 4. The conclusion is given in Section 5.

2. THE PROBLEM DESCRIPTION

The set of locations L contains the customers C and depots D. For each customer $i \in C$ we denote request r^i, time window $[t_a^i, t_b^i]$ and serving time t_s^i. Since we do not impose any specific request on the depot locations, we assume that some customers locations are appropriate, and assume $D \subseteq C$. However, we suppose that at specific depot a limited quantity of products W_d can be transferred during a time horizon $[T_a^d, T_b^d], d \in D$.

A fleet of homogenous vehicles V of capacity Q is available, where $Q \leq W_d, d \in D$. We denote by $R_v = (v^0, v^1, ..., v^m, v^0)$ a route of vehicle $v \in V$. With v_a^i, v_d^i we denote the arrival and the departure time at location v^i, respectively, and $q(v^i)$ represents the quantity of products in vehicle V departing from v^i. Let $dist(v)$ be the total distance of the route of vehicle v, and $n(v)$ the number of locations visited. With $e(s)$ we denote the number of routes starting (and ending) at depot $d \in D$.

A route R_v is feasible if the following holds:

- Starting and ending location is depot, $v^0 \in D$;
- Remaining locations are customer locations, $v^i \in C, i \in \{1,2,...,m\}$;
- All customers on a route are different, $v^i \neq v^j, i, j \in \{1,2,...,m\}, i \neq j$;
- Aggregated request of customers does not violate vehicle capacity constraint $q(v^0) \leq Q$;
- The time constraint at each depot is respected, $T_a^{v^0} \leq v_d^0 \leq v_a^0 \leq T_b^{v^0}$;
- Time constraint at each customer is respected, $v_a^i + t_s^i \leq t_d^i, v_d^i \leq t_d^i, i \in \{1,2,...,m\}$.

The distribution plan $\{R_v, v \in V\}$ is feasible if:

- Each route R_v is feasible;
- Each customer belongs to exactly one route;
- The aggregated demand of the routes starting at each depot does not violate the depot capacity constraint.

The goal of optimization is to find a feasible distribution plan with minimal distribution costs. The first priority is the number of depots, the second is the number of vehicles and the third is the total distance travelled.

3. ALGORITHM

In this section we present the algorithm for solving the location routing problem with time windows. First, we will describe the basic structure of the algorithm and the representation of the solution. Next, we present transformations defining the solution space and finally we describe the algorithm. As a local search procedure we choose Simulated annealing. The algorithm is divided into two main parts. Namely, we distinguish the goal of minimizing the number of depots and vehicles in the first part and minimizing the total distance during the second part.

The basic structure of the algorithm is *Node*. To each customer location we assign two types of nodes: *C-Node* that represents a customer and *D-Node* that represents a depot (we assumed in our test cases that C=D). We consider set of $2|V|+1$ lists. Lists $l^i, 1 \leq i \leq |V|$, are the singleton lists containing only one *D-Node*, while $l_j^i, 1 \leq i \leq |V|, 1 \leq j \leq |V|$, are the lists containing only *C-Nodes*. Each *Node* belongs to exactly one list, and the *Nodes* contained in the list with index $2|V|+1$ are not the part of the current solution.

With this structure of the solution we apply the following transformations: Opening a depot location, closing a depot location, and swapping two depot locations. Within the sequence of visited customers we apply the classical routing transformations move, swap, 2-opt and cross. For more details see [2]. We also apply the transformation of moving and swapping two routes.

The algorithm is divided into 3 phases:

- Phase 0 Building the feasible solution;
- Phase 1 Minimization of the number of depot locations and the number of vehicles;
- Phase 2 Minimization of the total distance.

The common evaluation function for all phases can be written in the following form:

$$E(R) = c_1 n(R) + c_2 \sum_{s \in S} e(s)^{\rho} + c_3 \ln(m^* + 1) + c_4 dist(R),$$

where $n(R)$ represents the number of routes, and $dist(R)$ represents the total route length.

During the Phase 0 and the Phase 1, the algorithm tends to move all customers from the particular vehicle (list). The number of customers on that vehicle is denoted by m^*. In the Phase 0, a particular vehicle is a dummy that contains all *C-Nodes* and all *D-Nodes*. Therefore, we set $c_1 = c_2 = 0$, and $c_3, c_4 > 0$. During the Phase 1, algorithm periodically and randomly chooses a route, and also tends to group routes to few depots. Accordingly we use, $c_1, c_2 >> c_3 >> c_4 > 0$. Finally in the last phase, algorithm tends to minimize the total distance so we set $c_1, c_2 >> c_4 > 0, c_3 = 0$.

4. COMPUTATIONAL RESULTS

The quality of the algorithm was tested on the several test instances generated on 500, 1000 and 2000 customers. The customer request was uniformly generated in $r^i \in (0,1200]$, and the time-windows $t_a^i \in [0,180]$, $t_d^i \in [360,600]$. Customer's locations are randomly chosen in the radius 10000. The capacity of depots are identical and set to W = 40000, and vehicle capacity is C = 10000.

As noted, the local search procedure is Simulated annealing. We apply the geometric cooling schedule scheme in the following way. The initial temperature is $T = T_0$, and $T := T^{\theta}$ after each τ seconds. The algorithm stops if $T < T_{end}$. Experimentally we set $\tau = 10/3$ seconds and $T_{end} = 0{,}1$.

The algorithm was coded in C++, and all tests were performed on Intel Pentium processor 1,6GHz. For each instance we run the algorithm 5 times and compare the best solution found with the average. There were no variation in the number of depots and the number of vehicles found. Therefore, we report best and average values for total distance. Among different cooling parameter θ, we report results for two values: $\theta_1 = 0{,}99$ and $\theta_2 = 0{,}995$. Table 1 summarizes the testing results for $\theta_1 = 0{,}99$.

Table 1. The results of testing for $\theta_1 = 0{,}99$.

Test Instance	Solution			
	$s(R)$	$n(R)$	*min dist*(R)	*avg dist*(R)
501	8	29	156962	162456
502	8	30	160392	163329
503	8	30	163381	167915
1001	12	46	224045	229894
1002	12	47	223667	229538
1003	12	46	225484	233060
2001	16	61	311867	316705
2002	16	61	302435	314825
2003	16	61	308875	319491

The average running time for the scheme $\theta_1 = 0{,}99$ is 1193 seconds and for $\theta_2 = 0{,}995$, 2292 seconds. The comparison of individual and best solution is summarized in Table 3.

In the Table 2 we summarize results for $\theta_2 = 0{,}995$.

Table 2. The results of testing for $\theta_2 = 0{,}995$.

Test Instance	Solution			
	$s(R)$	$n(R)$	*min dist*(R)	*avg dist*(R)
501	8	29	159144	161167
502	8	30	161432	164934
503	8	30	165158	168142
1001	12	46	221551	225215
1002	12	47	225566	232923
1003	12	46	219656	223648
2001	16	61	300738	306922
2002	16	61	303994	309822
2003	16	61	300129	307220

The deviation from best solutions for each cooling parameter ranges from 1,27% to 4,10%. On average, it is 2,88% for $\theta_1 = 0{,}99$, and 2,04% for $\theta_2 = 0{,}995$, and can be characterized as acceptable taking the problem complexity into account.

Table 3. The average deviation from best solution for $\theta_1 = 0{,}99$ and $\theta_2 = 0{,}995$.

Test Instance	The average deviation in %	
	$\theta_1 = 0{,}99$	$\theta_2 = 0{,}995$
501	3,50	1,27
502	1,83	2,17
503	2,90	1,81
1001	2,61	1,65
1002	2,63	3,26
1003	3,36	1,82
2001	1,55	2,06
2002	4,10	1,92
2003	3,44	2,36
Average	2,88	2,04

5. CONCLUSION

In this paper we have proposed the algorithm for solving the problem of locating depot locations in distribution networks. The algorithm runs in two phases respecting the optimization goals. Quality of the algorithm was tested on several large scale examples.

As noted, analyzed problem can be seen as a location routing problem with time windows. Testing the algorithm on classical location routing instances may give the insights for directions for further improvements of the algorithm.

In future research, we will analyze possibilities of exploring neighbourhoods with other procedures, e.g. via variable neighbourhood search. Also, reducing problem dimensions by clustering techniques may lead to significant improvements.

ACKNOWLEDGMENT

The work presented here was supported by the Serbian Ministry of Education and Science (projects III 44006 and OI 174018).

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