
MULTI-TRIP VEHICLE ROUTING WITH ORDER COMPATIBILITIES AND ORDER SCHEDULING: A PROBLEM ARISING FROM SUPPLY CHAIN MANAGEMENT

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***Abstract:** In this talk we introduce several new classes of rich vehicle routing problems which are motivated by a real-world problem arising in supply chain management. The problems are based on a specific 2-echelon distribution system where products come to the depot from different factories by semi-trailers and these semi-trailers are also used for short-haul distribution, i.e. the load from different factories is not consolidated at the depots. We specify the problem types and outline our algorithmic approaches used for solving.*

***Keywords:** multi-trip vehicle routing, order incompatibilities, supply chain management, meta-heuristic.*

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1. THE PROBLEM

In this talk we introduce several new classes of rich vehicle routing problems which are motivated by a real-world 2-echelon distribution system arising in supply chain management.

In our application we receive orders from customers for customized production of specific variants of a complex, i.e. voluminous and multipartite product. Depending on the variant these orders are produced in different factories. Deliveries of orders are planned for a planning horizon of T days.

Every order o has a certain volume $\text{vol}(o)$ and it is associated with a factory $f(o)$ and its customer location $\text{cust}(o)$. Distribution is done via a two-echelon channel using a set D of intermediate depots. Depending on the customer location every order o has a certain set $D(o)$ of admissible depots. At the factory (subsets of) orders are loaded onto trailers which are then transported to one of the depots by a factory truck. At the depot the trailer is decoupled and stored overnight. The next day the trailer is coupled to a depot truck which delivers the load on a route that is defined by the reverse loading sequence of the orders loaded on the trailer and which starts and ends at the depot. The factory truck returns to the factory with an empty trailer. Let F be the set of factories, D be the set of depots, $V(f)$ be the set of factory trucks available at factory f in F

and $V(d)$ the set of depot trucks available at depot d in D .

Trailers have a maximal volume capacity Q and the daily routes of the depot trucks have a maximum duration of Dur . Due to the volume of the orders (products) the number of orders which can be packed on a semi-trailer is rather small and the duration of depot routes are relatively short. Therefore a depot truck is able to perform several delivery tours a day. Now the problem is to determine an optimal distribution of a set of fixed orders from the factories to the customers and here minimizing the number of trucks is the primary objective while minimizing the distances travelled is the secondary objective. Thus in our problem we address the cost optimal movement of flows throughout the network from their origins to their destinations as well as the management of the fleets required to provide transportation.

The distribution involves several decisions:

- we have to determine which orders should be distributed on which day (order scheduling),
- for every day and every factory the selected orders have to be assigned to depots (channel selection),
- all orders assigned to the same depot have to be assigned to trailers (clustering),

- the loading sequence of the orders has to be determined since this sequence determines the delivery route from the depot (routing),
- the trailer loads at the depots have to be bundled to multi-trip routes for depot trucks (bundling).

All these decisions are interconnected and have to be done under the objective to first minimize the total number of trucks needed and second to minimize the total driving distance. Thus in its entire complexity the problem is a multi-depot multi-trip vehicle routing problem with order (in-) compatibilities for trips and order scheduling.

The distribution system is illustrated in Figure 1.

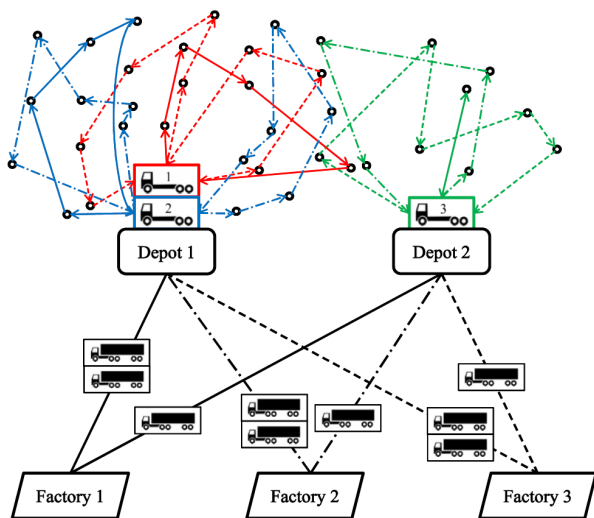


Figure 1. Example of the distribution system

Assume that we have constructed (for every factory) the set of feasible trailer loads, i.e. those assignments of orders to trailers and depots which lead to routings for which maximum load volume is obeyed. Then the problem can be modelled as a set-partitioning problem with the decision variable

$$x(l,h,k,t) = 1 \text{ if load } l \text{ is delivered by factory truck } h \text{ in } V(f) \text{ to depot } d \text{ and by depot truck } k \text{ in } V(d) \text{ on day } t \text{ in } T, 0 \text{ and otherwise.}$$

Although multi-echelon distribution systems have been studied already in the 1980s, see [7], this problem domain has become recognition and importance with the emergence of complex supply chain management concepts integrating the procurement, production and distribution processes. In the applications discussed in literature, as for instance in City Logistics [2], a central aspect is the possibility of consolidating the freight at the depot. In our application such a consolidation which requires unloading and reloading of at least a part of the cargo is not possible. All semi-trailers remain

untouched and the load is delivered by a depot truck in the order given by the loading sequence at the factory. Thus the entire routing is already determined at the factory. The depots do not perform any warehousing activities and do not require complex infrastructure for handling, i.e. the depots are parking lots. This resembles the satellite concept of specific city logistic approaches.

Since cargo can only be consolidated on the level of complete semi-trailers, planning requires the synchronization of all semi-trailer routes to a set of multi-trip routings for the depot trucks.

At its core our distribution problem can be viewed as a multi-trip vehicle problem (MTVRP) as studied in [1], yet, with a specific additional synchronization constraint and a fixed cost term for every single trip accounting for the traveling distance between factory and depot. Here the synchronization constraint requires that only orders of the same factory can be clustered into one trip, i.e. there is a specific type of incompatibilities between orders disallowing joint distribution. Order or product incompatibilities are known from vehicle routing problems with compartments (VRPC) where vehicles with compartments are employed in order to allow transporting inhomogeneous goods together on the same vehicle, but in different compartments (see [3]). Thus our problem can be viewed as a combination of MTVRP and VRPC. Therefore we denote such problems as multi-depot multi-trip vehicle routing problems with (in-) compatibilities (MDMTVRPC).

Note that the underlying real world problem in this study requires several additional synchronization constraints which lead to more complex RVRP variants.

For the general problem with order scheduling given in our application an additional synchronization constraint requires a balanced use of trailers, i.e. the number of loads/trailers transported to a depot has to be equal over the week such that the trailers used on day t for transporting a load from a factory to a depot are transported back from the depot to the factory on day $t+2$ and are available for another load on day $t+3$.

Our problem is quite complex and it can be relaxed in several aspects. Setting $T=1$ we obtain a model for a daily planning approach. If $|D(o)|=1$ for every order the problem decomposes into a number of (single depot) multi-trip problems (MTVRPC). On the other hand the problems can be extended by time window constraints. In the (MD)MTVRPC with time windows (TW) we have to find for every depot a set of trips such that time window constraints at the customers are met and two trips

that overlap in time cannot be served by the same truck.

2. THE SOLUTION APPROACH

In former research on several complex rich vehicle routing problems we have experienced that heuristic approaches combining local search (LS) and large neighborhood search (LNS) easily improve the individual methods and we could show that concurrent LS/LNS neighborhood search (CNS) which combines LS and LNS moves concurrently is highly effective and efficient for solving RVRP variants (see [5]). In [4] we show that this approach outperforms variable neighborhood search (VNS) developed in [8] if the same portfolio of moves is used.

In [5] we describe the implementation of a general CNS software framework allowing customizing appropriate solvers for different classes of rich vehicle routing problems. In our research reported here we have built upon and extended this framework.

LS is based on moves/neighborhoods which allow only small modifications to the current solution and thus supports intensification. The framework contains generic implementations for the following set of moves which have shown to be effective in many LS-approaches for solving different RVRP presented in literature: Relocate, RelocateI, Exchange, 2-opt and 2-opt*. LNS (see [9]) implements the ruin-and-recreate principle by combining different removal and insertion operations by which orders are removed from the solution (their tours) first and then reinserted again into the solution. Thus LNS supports diversification. In our framework we combine Random removal, Worst removal and Shaw removal with Greedy insertion and Regret n (with $n=2, \dots, 5$) insertion. For a detailed description of the LS- and LNS-moves see [5] and [9], respectively.

The acceptance of a neighbour generated by a move is decided by a metaheuristic strategy which guides the search and prevents (early) termination in a (bad) local optimum. In our framework we have implemented record to record travel (RRT) a deterministic annealing techniques and the attribute based hill climber (ABHC) a specific variant of tabu search (TS).

In RRT (see [6]) randomly selected neighbours are accepted if they are not worse than the best solution found so far by a prespecified relative deviation.

In tabu search entire neighbourhoods are scanned and a move to the best neighbour is performed even

if it does not lead to an improvement. In order to prevent cycling solutions are temporarily declared tabu for a number of iterations. ABHC (see [11]) is a parameter-free TS-variant. It uses a generic attribute concept for specifying non-tabu neighbors, which has to be specialized for every problem domain. Then a solution is acceptable if it is the best solution visited so far for at least one attribute that it possesses.

Based on these concepts we have developed two generic concurrent approaches: CNS-ABHC and CNS-RRT. In both methods we start with the construction of an initial solution in a first phase followed by an improvement phase which is applied until a predefined time or iteration limit is reached. In each iteration of the improvement phase we decide randomly which type of neighborhood, i.e. LS or LNS, to use. Then the resulting neighbor is accepted using either metaheuristic control. Specifying a probability parameter p^{LS} the selection is biased towards either LS or LNS. In LNS we apply a fast 2-opt steepest descent improvement to all modified routings. Figure 2 displays the general logic of CNS.

The framework provides a complete and ready-to-use solver suite for the standard capacitated vehicle routing problem offering several mechanism (templates) to be adapted or extended according to the vehicle routing problem variant to be solved. Thus when solving a specific VRPs like the ones discussed here one has for instance to modify the moves such that the specific constraints, i.e. route duration, order compatibilities etc. are obeyed.

3. COMPUTATIONAL EXPERIENCE

Since the MTVRPC is a new VRP class there is no set of benchmark instances in the literature which we can use for our computational tests. Therefore we have generated benchmark instances for the MTVRPC, the MTVRPCTW and the MDMTVRPCTW based on the instance set of Solomon [10] for the VRPTW. These benchmark problems assign geographical coordinates to customers, i.e. distances are euclidean. For our problem we assume that each customer requires the delivery of exactly one order.

On these instances we have compared the two CNS-implementations with a basic LNS-RRT implementation. Note, that we have customized the specific approaches while using the standard parametrization identified in [5] to be appropriate for the VRP. Our computational tests were performed on an Intel Xeon E5430 2.66 GHz PC with eight cores and operating system Microsoft Windows 7.

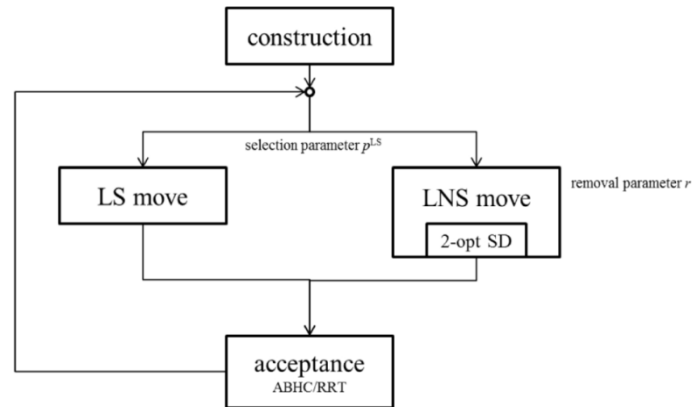


Figure 2. Logic of CNS

The computational results are as expected:

- cost (total traveling distance) for MTVRPCTW increases compared to MTVRPC, yet,
- cost decreases again for MDMTVRPCTW.
- LNS-RRT and CNS-RRT are converging much faster than CNS-ABHC, but
- CNS-ABHC is able to find significantly better solutions under more running time.
- CNS-RRT and LNS-RRT perform relatively similar, yet, CNS-ABHC is able to reduce the fleet size.
- With increasing complexity of MTVRPCTW and MDMTVRPCTW all approaches need longer running times to the point of convergence, but again CNS-ABHC is able to find better solutions consistently.
- Especially, for the MDMTVRPCTW, with RRT one is able to find solutions with shorter total traveling distances (cost) than ABHC, but the number of trucks is smaller for ABHC.

We attribute the last property to the higher potential of diversification under ABHC in comparison to RRT. ABHC is able to accept moves that increase the objective function value significantly and thus ABHC is able to steer the search into areas of the solution space that under RRT are not in reach.

4. CONCLUSION

After all, we could show that as for many other VRP-classes (see [4] and [5]) CNS is a rather effective and efficient strategy for the problem types described in this paper, too. Thus from this feasibility study we can expect that instances of the specific real world 2-echelon problem which has been the motivation of the study can be solved by

our approach satisfying the requirements of effectiveness (solution quality) and efficiency (solution time).

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