OPEN TRAVELING SALESMAN PROBLEM WITH TIME WINDOWS

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***Abstract**: Routing and scheduling of vehicles are important operational issues in the logistic management. This paper deals with the open traveling salesman problem with time windows (OTSPTW). The OTSPTW is an extension of classical traveling salesman problem that is well known in optimization. The goal of the OTSPTW is to find optimal shortest route (in time or distance units) for a vehicle with unlimited capacity in order to serve a given set of customers. The difference of the OTSPTW from the classical TSP model is that the vehicle do not need to return to the depot after a service of the last customer and all the customers need to be served between a given time interval (time window). In this paper, we formulate a mathematical model to capture all aspects of that problem based on mixed integer programming (MIP) with linear objective function and constraints and we present the source code for General Algebraic Modeling System (GAMS).*

Keywords: *Open Traveling Salesman Problem with Time Windows, Mixed Integer Programming, General Algebraic Modeling System.*

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1. INTRODUCTION

Logistics is evidently one of the major dimensions of many firms and the related logistic costs constitute a significant share of the total costs of every organization. Many variants of routing and scheduling problems that can be very rewarding are known in the field of logistics. The well-known traveling salesman problem (TSP) is one of the most discussed problems in operation research. The importance of that problem is evidently seen in the field of operation research or artificial intelligence etc., because of its computational complexity, but also the importance follows from a great practical applicability, that why the problem can be applied in more general way.

This paper describes the open traveling salesman problem (OTSP). The OTSP is a popular problem in the field of distribution management and can be used to model many real-life problems. For example if the company deals without its own vehicle fleet and has to hire some vehicle to deliver its products to customers. In this case, the company is not concerned whether the vehicle returns to the depot, and does not pay the traveling costs between the last customer and the depot. The OTSP can be described as follows: Consider a depot from that some products have to be delivered to a set of customers. Products are loaded on a vehicle at the depot and they are transported to the customers. The capacity of vehicle is unlimited, so that all customers' demand can be satisfied with the one route of the vehicle. As the vehicle is hired, after end of a route, vehicle does not need to return to the depot and the delivery process is terminated as soon as the final customer is served. We assume that the shortest distance between depot and each customer's location is known, as well as between each pairs of customer's location is known. The goal is to find optimal shortest route for a vehicle.

The practical problems of physical distribution often include the need to respect the time restriction. Frequently we consider time restriction that are a consequence of first possible time of service, the last acceptable time of service or the need to serve during the given time interval. The above mentioned terms are known as time windows. So we can talk about the open travelling salesman problem with time windows (OTSPTW). If it is necessary to consider only the first possible time of service or the last acceptable time of service, the problem is known as problem with soft delivery time windows, if we are dealing with time interval with given lower and upper limit, those problems are known as problems with hard delivery time windows, e. g. [5]. Another description of soft windows can be found e.g. in [3], [8], [9], where the violation of time restriction is allowed, although incurring some cost. In formulation of that problem we consider that the shortest travel time between depot and each customer's location is known, as well as between each pairs of customer's location is known.

2. THE OPEN TRAVELING SALESMAN PROBLEM

The OTSP can be stated as follows: Consider a network with n nodes. Indices i and j refer to customers and take values between 2 and n, while index i = 1 refers to the depot. Also there is a shortest distance d_{ij}, i,j = 1, 2, ... n associated with the each pair of customers and also with the each customer and the depot. The goal is to find a shortest route for a vehicle with an unlimited capacity that served all the customers so that the route ends after the final customer is served. Mathematical programming formulation of OTSP requires two type of variables: the binary variables x_{ij}, i,j = 1, 2, ... n with a following notation: x_{ij} = 1 if customer i precedes customer j in a route of the vehicle and x_{ij} = 0 otherwise. Further on, we will apply the variables u_i, i = 2, 3,…n that based on well-known Tucker's formulation of the traveling salesman problem:

Then, the mathematical model for OTSP is:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \tag{1}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 2,3,...n \;\; i \neq j \tag{2}$$

$$\sum_{j=2}^{n} x_{ij} \leq 1 \quad i = 1,2,...n \;\; i \neq j \tag{3}$$

$$\sum_{j=2}^{n} x_{1j} = 1 \quad i = 2,3,...n \tag{4}$$

$$u_i - u_j + n x_{ij} \leq n-1 \quad i,j = 2,3,...n \;\; i \neq j \tag{5}$$

$$x_{ij} \in \{0,1\} \quad i,j = 1,2,...n \;\; i \neq j \tag{6}$$

The objective function (1) models the total distance of vehicle route. Constraints (2) ensure that the vehicle enters to every customer and constraints (3) ensure that the vehicle does not need to depart from every customer, because the route ends after serving the last of them. Constraints (4) ensure that the vehicle starts its route exactly once and constraints (5) avoid the presence of sub-tour.

Source code for GAMS (General Algebraic Modelling System) for solving of OTSP is as follows:

```
$title Open Travelling Salesman Problem
$ontext
Input data
$offtext
Sets
i /1*n/
subi(i) /2*n/
alias (i,j)
alias (subi,subj);
Sets offdiag1(i,j)
    offdiag2(i,j);
offdiag1(i,j)=yes;
offdiag1(i,i)=no;
offdiag2(i,j)=offdiag1(i,j);
offdiag2(i,'1')=no;
* Matrix of shortest distances (diag. elements = 0)
Table d(i,j)
;
$ontext
Mathematical model
$offtext
Scalar n;
n=card(i);
Variables f, u(j);
u.fx('1')=0;
u.lo(subi(i))=1;
u.up(subi(i))=n;
Binary variables x(i,j);
Equations
ohr1(i), ohr2(j), anti(i,j), ohr3, ucel;
ucel.. f=e=sum((i,j),d(i,j)*x(i,j));
ohr1(i).. sum(subj(j),x(i,j)$offdiag1(i,j))=l=1;
ohr2(subj(j)).. sum(i,x(i,j)$offdiag1(i,j))=e=1;
ohr3.. sum(subj(j),x('1',j)=e=1;
anti(i,j)$offdiag2(i,j).. u(i)-u(j)+n*x(i,j)=l=n-1;
Model otsp /all/;
Solve otsp using mip minimizing f;
Display x.l;
```

3. THE OPEN TRAVELING SALESMAN PROBLEM WITH TIME WINDOWS

Further on, we will consider the time window for each customer, so that there is known a given time interval $\langle e_i ; l_i \rangle$, where e_i represents the first possible time to serve i-th customer and l_i represents the last acceptable time to leave the i-th customer, i = 2, 3,…n. Also, there is known a service o_i time of each customer, i = 2, 3,…n. The variable w_i, i = 2, 3,…n, represents a possibility of waiting by a next customer if a service is not allowed. Parameters d_{ij}, i,j = 1, 2, ... n associated with the each pair of customers and also with the each customer and the depot represent the shortest travel times.

Thus, the mathematical models can be stated as follows:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}x_{ij} + \sum_{i=2}^{n} o_i + \sum_{j=2}^{n} w_j + v\sum_{i=2}^{n} u_i \quad (7)$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1 \quad j = 2,3,...n \;\; i \neq j \quad (8)$$

$$\sum_{j=2}^{n} x_{ij} \leq 1 \quad i = 1,2,...n \;\; i \neq j \quad (9)$$

$$\sum_{j=2}^{n} x_{1j} = 1 \quad i = 2,3,...n \quad (10)$$

$$u_i + o_i + w_j + d_{ij} - M\left(1 - x_{ij}\right) \leq u_j \quad i = 1,2,...n \;\; j = 2,3,...n \;\; i \neq j \quad (11)$$

$$e_i \leq u_i \quad i = 2,3,...n \quad (12)$$

$$u_i + o_i \leq l_i \quad i = 2,3,...n \quad (13)$$

$$u_1 = 0 \quad (14)$$

$$o_1 = 0 \quad (15)$$

$$x_{ij} \in \{0,1\} \quad i,j = 1,2,...n \;\; i \neq j \quad (16)$$

$$w_j \geq 0 \quad j = 2,3,...n \quad (17)$$

where M – represents a large positive number

As mentioned earlier, constraints (8) ensure that the vehicle enters to every customer and constraints (9) ensure that the vehicle does not need to depart from every customer, because the route ends after serving the last of them and constraints (10) ensure that the vehicle starts its route exactly once. Constraints (5) is replaced by constraints (11) that is there not only to avoid the presence of sub-tour, but also to ensure the calculation of the real starting time of service for each customer (variables u_i) and concurrently with the use of constraints (12) and (13) ensure that all the time windows are met. Constraint (14) denotes that the service begins in zero service time and constraint (15) denote the zero service time in the depot. The objective is calculated as a sum of total time, the time of waiting and the time of service. The formula $v\sum_{i=2}^{n} u_i$ in the objective function ensures the calculation of the lowest values of variables u_i., without this formula that model can returns alternative solution of variables u_i that do not ensure the calculation of real starting of service of customers (parameter v is a weight, usually a small number). The total time of service can be calculated by subtracting the weighted value of the sum of u_i from the value of objective function (7).

Sorce code for GAMS for solving of OTSPTW is as follows:

```
$title Open Traveling Salesman Problem with Time Windows
$ontext
Input data
$offtext
Sets
i /1*n/
subi(i) /2*n/
alias (i,j)
alias (subi,subj);
Sets offdiag1(i,j)
    offdiag2(i,j);
offdiag1(i,j)=yes;
offdiag1(i,i)=no;
offdiag2(i,j)=offdiag1(i,j);
offdiag2(i,'1')=no;
Scalars M /M/
 v /v/;
* Matrix of shortest travel times (diag. elements = 0)
Table d(i,j)
;
Parameters
e(j) / /
l(j) / /
o(j) / /;
$ontext
Mathematical model
$offtext
Scalar n;
n=card(i);
Binary variables x(i,j);
Positive variables w(i);
Free variable f,u(i);
u.fx('1')=0;
Equations
ohr1(i), ohr2(j), ohr3, ohr4(i,j), ohr5(j), ohr6(j), ucel;
ucel..
f=e=sum((i,j),d(i,j)*x(i,j))+sum(subi(i),o(i))+sum(subi(i),
w(i))+v*sum(subi(i),u(i));
ohr1(i).. sum(subj(j),x(i,j)$offdiag1(i,j))=l=1;
ohr2(subj(j)).. sum(i,x(i,j)$offdiag1(i,j))=e=1;
ohr3.. sum(subj(j),x('1',j)=e=1;
ohr4(i,j)$offdiag2(i,j)..    u(i)+o(i)+w(j)-u(j)+d(i,j)-M*(1-
x(i,j))=l=0;
ohr5(subj(j)).. u(j)+o(j)=l=l(j);
ohr6(subj(j)).. u(j)=g=e(j);
```

```
Model otsptw /all/;
Solve otsptw using mip minimizing f;
Display x.l,u.l,w.l;
```

4. CONCLUSION

In this paper, we have considered a relative new problem, the open traveling salesman problem with time windows, which addresses practical issues in routing vehicle fleets. The problem was formulated on the base of mixed integer programming (MIP) with linear objective function and constraints. So that formulation allows the use of standard software for solving MIP problems.

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