THE VEHICLE REROUTING PROBLEM WITH TIME WINDOWS AND SPLIT DELIVERY

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Abstract: *Many logistics company use the same set of vehicle routes every day. Unexpectedly high customers' demand in one or more nodes can cause a disruption in vehicle operations. Disruption occurs since, for one or more planned routes, capacity constraints can be broken. In such a situation, dispatchers must decide how to organize a distribution in the way to minimize negative effects of disruptions. The mathematical programming formulation for the vehicle rerouting problem in the case of split delivery is proposed in the paper. Performed numerical experiments show that split delivery concept of distribution, in the case of disruptions in vehicle operations, outperform distribution approaches without split delivery.*

Keywords*: Vehicle routing problem, disruptions, split delivery*

1. INTRODUCTION

Many logistics companies, on a daily basis, deliver various goods to the customers. In many cases, companies deliver goods to the same set of the customers every day. In such a situation, dispatchers usually decide to use the same set of vehicle routes every day. The benefits of performing the same vehicle routes on a daily basis are obvious. During the time, drivers become familiar with the routes where they make deliveries. It is also easier for companies to plan fleet size and fleet usage. Simultaneously, the costs can be planned for a longer period of time.

Sudden changes in customer demands can cause situations where the planned vehicle routes are not the best possible. Significantly increased customer demands can make that some vehicle capacity constraints are broken. The disruptions in vehicle operations can produce the following negative effects: there is a need to engage additional vehicles; the delivery cost could be increased; some customers cannot be served, etc.

We consider in this paper the case when split delivery is allowed in the situation when disruption occurs. We propose, for this case, the mix-integer linear programming mathematical formulation. We clearly show that the split delivery can significantly decrease negative consequences of disruptions.

The paper is organized in the following way. The literature review and statement of the problem are given in the Section 2. Mathematical formulation of the problem is described in the Section 3. Numerical experiments are presented in the Section 4. Finally, the conclusions and directions for future research are given in the Section 5.

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2. LITERATURE REVIEW AND STATEMENT OF THE PROBLEM

The area of disruption management has been studied since 1984 (Teodorović and Guberinić, 1984). In distribution systems different disruptions can occur, such as: vehicle breakdown, changes in customer demand, traffic jam, changes in delivery locations, etc. There are several papers in the literature devoted to the vehicle rerouting problems. Li et al. (2009a,b) considered the vehicle routing problem which arise when one or more vehicles have breakdown. The authors proposed insertion heuristic based on Lagrangean relaxation. Mu et al. (2011) considered similar problem and proposed model based on Tabu search metaheuristic. Wang et al. (2011,2012) proposed mathematical formulation as well as, heuristic algorithms for the case when few disruption events occur. Hu and Sun (2012) proposed knowledge-based approach to mitigate disruptions during distribution process, while Hu et al. (2013) proposed local search algorithms and object-oriented modeling. Mu and Eglese (2013) analyzed the problem when the delivery to central warehouse is in delay. The authors proposed the model based on Tabu search metaheuristic.

We consider the case when companies use the same set of vehicle routes every day. The problem arises when the customers significantly increase demand and consequently, some vehicle routes become infeasible. This problem was studied by Spliet et al. (2014) and Nikolić and Teodorović (2015). Spliet et al. (2014) based their model on the capacity vehicle routing problem. They proposed the mathematical formulation, as well as heuristic algorithm. Nikolic and Teodorović (2015) studied the vehicle rerouting problem in the case of time windows and proposed multi-objective mathematical formulation.

In this paper, we further expand the research performed by Nikolić and Teodorović (2015). We consider the case when split deliveries are allowed. In other words, we consider the case when nodes could be served by more than one vehicle.

3. MATHEMATICAL PROGRAMMING FORMULATION

Let us denote by $G=(N,A)$ an oriented graph, where V is a set of nodes $[V=\{0,1,\dots,n,n+1\}]$, and A is a set of edges $[(i,j)\in A]$. We denote by 0 and n+1 the central depot.

We also introduce the following notation:

- Binary decision variables:

$$\xi_i=\begin{cases}1, & \text{if node } i \text{ will not be served}\\ 0, & \text{otherwise.}\end{cases}$$

$$z_{ik}=\begin{cases}1, & \text{if the vehicle } k \text{ will make delivery to node } i\text{, although this node is not in its original route}\\ 0, & \text{otherwise.}\end{cases}$$

$$y_{ij}^k=\begin{cases}1, & \text{if vehicle } k \text{ travels from node } i \text{ to node } j\\ 0, & \text{otherwise.}\end{cases}$$

- The other decision variables are:

w_i^k - time when start service at node *i* by vehicle *k*

f_{ik} – the fraction of demand of node *i* delivered by vehicle *k*

- The following are the input values:

x_{ij}^k - is equal 1 if in the initial set of routes vehicle *k*, after servicing node *i*, goes to node *j*

w_i - importance (weight) to serve the node i

u_i – importance (weight) that node i will not be serviced in the route that is not the original route of the node i

c_{ij} – the cost for travel from node i to node j

q_i – demand at node i

Q – vehicle capacity

t_{ij} – travel time from node i to node j

s_i – duration of service at the node i

M – large positive number

Mathematical formulation of the vehicle rerouting problem in the case of split delivery, based on Nikolić and Teodorović (2015) and Ho and Haugland (2004), can be given in the following way:

$$\min \quad F_1 = \sum_{i \in N} w_i \, \xi_i \tag{1}$$

$$\min \quad F_2 = \sum_{k \in K} \sum_{i \in N} u_i \, z_{ik} \tag{2}$$

$$\min \quad F_3 = \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} \, y_{ij}^k \tag{3}$$

subject to:

$$\sum_{k \in K} \sum_{j \in \delta^+(i)} y_{ij}^k + \xi_i \ge 1 \quad \forall \, i \in N \tag{4}$$

$$\sum_{j \in \delta^+(0)} y_{0j}^k = 1 \quad \forall \, k \in K \tag{5}$$

$$\sum_{i \in \delta^-(j)} y_{ij}^k - \sum_{i \in \delta^+(j)} y_{ji}^k = 0 \quad \forall \, k \in K, \, j \in N \tag{6}$$

$$\sum_{i \in \delta^-(n+1)} y_{i,n+1}^k = 1 \quad \forall \, k \in K \tag{7}$$

$$w_j^k \ge w_i^k + s_i + t_{ij} - M\left(1 - y_{ij}^k\right) \quad \forall \, k \in K, (i,j) \in A \tag{8}$$

$$a_i \le w_i^k \le b_i \quad \forall \, k \in K, i \in V \tag{9}$$

$$\sum_{i \in N} g_i \, f_{ik} \le Q \quad \forall \, k \in K \tag{10}$$

$$\sum_{k \in K} f_{ik} = 1 \quad \forall \, i \in N \tag{11}$$

$$\sum_{j \in \delta^+(i)} y_{ij}^k \ge f_{ik} \quad \forall \, i \in N, k \in K \tag{12}$$

$$\sum_{i \in \delta^-(j)} \left(y_{ij}^k - x_{ij}^k\right) \le z_{jk} + \xi_j \quad \forall \, k \in K, \, j \in N \tag{13}$$

$$\xi_i \in \{0,1\} \quad \forall \, i \in N \tag{14}$$

$$z_{ik} = \{0,1\} \quad \forall\, k \in K, i \in N \tag{15}$$

$$y_{ij}^{k} \in \{0,1\} \quad \forall\, k \in K, (i,j) \in A \tag{16}$$

The objective function (1) that should be minimized represents the total number of unsatisfied customers. The total number of customers that will be served in the routes which are not their original routes is calculated by objective function (2). This objective function should be minimized. The objective function (3) that should be minimized represents the total travel costs.

Constraints (4) guarantee that if node *i* will not be served, the decision variable ξ_i must take value 1. All vehicles leave the depot, visit some nodes and return to the depot. This is guaranteed by the constraints (5), (6) and (7). Time windows must be satisfied according to the constraints (8) and (9). Constraints (10) and (11) guarantee that the vehicle capacity will not be exceeded by the total demand that will be delivered through the route. Because of constraint (12) if node *i* will not be served by the vehicle *k* , the decision variable f_{ik} must take value 0. If the vehicle *k* makes the delivery to the node *j*, and node *j* is not included in the initial route of the vehicle *k*, then variable z_{jk} will take the value 1 according to constraints (13). Constraints (14), (15) and (16) define decision variables as binary.

4. NUMERICAL EXAMPLES

To evaluate the effects of allowed split delivery, we have used benchmark examples, based on C101 Solomon benchmark example, given in (Nikolić and Teodorović, 2015). Nikolić and Teodorović, 2015 used lexicographic method to solve the problem instances. Their results, related to the case when the split delivery is not allowed, are given in Table 1. The results are given for the cases when the number of customers are 25 and 50 respectively. All instances have been solved by the CPLEX software.

Table 1. The results obtained when split delivery is not allowed (Nikolić and Teodorović, 2015)

The number of nodes	Increase in demand	C101					
		Fleet size: 3			Fleet size: 4		
		F1	F2	F3	F1	F2	F3
25 nodes	10%	0	1.12	249.00	0	1.12	249.00
	20%	0	1.15	239.33	0	1.13	251.92
	30%	1.02	1.15	239.27	0	2.16	271.29
	40%	1.30	2.22	257.76	0	3.27	272.84
	50%	2.34	1.12	243.47	0	3.29	273.30
	60%	3.60	1.09	220.37	0	4.31	265.00
	70%	4.78	0	185.81	1.04	3.37	271.48
	80%	4.78	0	185.81	1.04	3.37	271.48
	90%	6.04	2.21	192.02	2.30	5.55	284.41
	100%	6.04	2.21	192.02	2.30	5.55	284.41
50 nodes	10%	0	2.43	480.79	0	2.43	480.79
	20%	0	3.68	469.92	0	3.66	488.66
	30%	0	6.01	496.3	0	6.00	505.1
	40%	1.04	7.23	563.66	0	7.16	556
	50%	2.62	7.5	492.68	0	8.59	527.25
	60%	4.92	8.99	610.38	1.04	11.16	552.46
	70%	7.54	6.63	516.52	3.46	9.04	561.14
	80%	7.54	6.63	516.52	3.46	9.04	561.14
	90%	10.34	9.16	613.72	5.78	12.18	584.09
	100%	10.34	9.16	613.72	5.78	12.18	584.09

In this paper we have solved all instances considered by Nikolić and Teodorović (2015). We also use Lexicographic method and the CPLEX software. The obtained results are given in Table 2. In the case when there are one central depot and 25 customers, when the fleet size is equal 3 vehicles, for the six instances we obtained the better results than in the previous research. In the case when the fleet size is equal 4 vehicles, we obtained better results in two cases. Similar results are obtained for the cases where there are one central depot and 50 customers. In these cases, when the fleet size is equal 6 vehicles, we obtained better results for six instances, while in the cases where the fleet size is equal 7 vehicles, three times we discovered the better solution. The better results are denoted by bold and italic letter in Table 2.

Table 2. The results obtained in the case when split delivery is allowed

The number of nodes	Increase in demand	C101					
		Fleet size: 3			Fleet size: 4		
		F1	F2	F3	F1	F2	F3
25 nodes	10%	0	1.12	249.00	0	1.12	249.00
	20%	0	1.15	239.33	0	1.13	251.92
	30%	***0***	***2.21***	***257.28***	0	2.16	271.29
	40%	***1.3***	***2.18***	***256.60***	0	3.27	272.84
	50%	***2.3***	***3.37***	***253.52***	0	3.29	273.30
	60%	***3.46***	***2.38***	***288.07***	0	4.31	265.00
	70%	***3.66***	***2.07***	***229.53***	***0***	***4.39***	***278.66***
	80%	4.78	0	185.81	1.04	3.37	271.48
	90%	***5.16***	***3.2***	***258.49***	***1.3***	***6.55***	***304.73***
	100%	6.04	2.21	192.02	2.3	5.55	284.41
50 nodes	10%	0	2.43	480.79	0	2.43	480.79
	20%	0	3.68	469.92	0	3.66	488.66
	30%	0	6.01	496.30	0	6.00	505.10
	40%	***1.02***	***7.21***	***531.07***	0	7.16	556.00
	50%	***2.30***	***7.59***	***567.93***	0	8.59	527.25
	60%	***3.66***	***8.96***	***600.50***	***0***	***12.14***	***590.62***
	70%	***5.78***	***8.36***	***514.64***	***1.30***	***11.12***	***540.34***
	80%	7.54	6.63	516.52	3.46	9.04	561.14
	90%	***9.08***	***9.13***	***669.01***	***4.78***	***12.68***	***559.52***
	100%	***10.34***	***8.01***	***615.03***	5.78	12.18	584.09

5. CONCLUSIONS

Many logistics companies use the same set of vehicle routes every day for delivery goods to the customers. These routes sometimes can become unfeasible in the cases of very high customers demand.

When solving vehicle rerouting problem, we proposed and analyzed the split delivery concept. It has been shown, that by allowing the split deliveries the negative consequences of the disturbances can be significantly reduced.

We will explore various heuristic approaches in the future research that will enable us to attack the vehicle rerouting problems of big dimensions.

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