A MODELING APPROACH OF DESIGNING AND MANAGING SUPPLY NETWORKS

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Abstract: *Supply chain management is more and more affected by network and dynamic business environment. The fundamental decisions to be made during the design phase are the location of facilities and the capacity allocated to these facilities. An approach to designing sustainable supply networks is to develop and solve a mathematical programming model. The sustainability of supply networks can be measured by multiple objectives, such as economic, environmental, social, and others. Traditional concepts of optimality focus on valuation of already given systems. New concept of designing optimal systems is applied. Searching for a better portfolio of resources leads to a continuous reconfiguration and reshaping of systems boundaries. To respond to rapidly changing market conditions, firms must be able to dynamically form and dissolve business interactions. Using double combinatorial auctions can solve the problem. The proposed combined approach is promising for designing and managing supply networks.*

Keywords*: Supply network, De Novo optimization, multiple criteria, combinatorial auctions*

1. INTRODUCTION

Supply chain management has generated a substantial amount of interest both by managers and researchers and is more and more affected by network and dynamic business environment. The Supply network is defined as a system of suppliers, manufacturers, distributors, retailers and customers where material, financial and information flows connect participants in both directions (see for example Fiala, 2005). There are many concepts and strategies applied in designing and managing supply networks. The sustainability of supply networks can be measured by multiple objectives, such as economic, environmental, social, technological, and others.

Traditional concepts of optimality focus on valuation of already given systems. Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. New concept of designing optimal systems was proposed (Zeleny, 1990). As a methodology of optimal system design can be employed Multi-objective De Novo linear programming (MODNLP) problem for reshaping feasible sets in linear systems. The approach is based on reformulation of MOLP problem by given prices of resources and the given budget and searching for a better portfolio of resources. The paper presents approaches for solving the MODNLP problem for design of sustainable supply networks.

How to coordinate the decentralized supply network to be efficient as the centralized one? There are many concepts and strategies applied in managing supply networks. Auctions are

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important market mechanisms for the allocation of goods and services. An auction provides a mechanism for negotiation between buyers and sellers. Combinatorial auctions are those auctions in which bidders can place bids on combinations of items (see Cramton et al., 2006). In forward auctions a single seller sells resources to multiple buyers. In a reverse auctions, a single buyer attempts to source resources from multiple suppliers, as is common in procurement. Auctions with multiple buyers and sellers are called double auctions. Auctions with multiple buyers and sellers are becoming increasing popular in electronic commerce. We propose to use a model of double combinatorial auctions as trading model between layers of supply network.

2. SUSTAINABLE SUPPLY NETWORKS

In the next part a sustainable supply network design problem is formulated. The fundamental decisions to be made during the design phase are the location of facilities and the capacity allocated to these facilities. An approach to designing a sustainable supply network is to develop and solve a mathematical programming model. The mathematical program determines the ideal locations for each facility and allocates the activity at each facility such that the multiple objectives are considered and the constraints of meeting the customer demand and the facility capacity are satisfied. A general form of the model for the sustainable supply network design is Multi-objective linear programming (MOLP) model. A specific model is presented below.

The model of a supply network consists of 4 layers with m suppliers, S_1, S_2, ... S_m, n potential producers, P_1, P_2, ... P_n, p potential distributors, D_1, D_2, ... D_p, and r customers, C_1, C_2, ... C_r. The following notation is used:

- ai = annual supply capacity of supplier i, bj = annual potential capacity of producer j,
- wk = annual potential capacity of distributor k, dl = annual demand - customer l,
- f_j^P = fixed cost of potential producer j, f_k^D = fixed cost of potential distributor k,
- c_{ij}^S = unit transportation cost from Si to Pj, c_{jk}^P = unit transportation cost from Pj to Dk,
- c_{kl}^D = unit transportation cost from Dk to Cl, e_{ij}^S = unit environmental pollution from Si to Pj,
- e_{jk}^P = unit environmental pollution from Pj to Dk, e_{kl}^D = unit environmental pollution from Dk to Cl,
- x_{ij}^S = number of units transported from Si to Pj, x_{jk}^P = number of units transported from Pj to Dk,
- x_{kl}^D = number of units transported from Dk to Cl,
- y_j^P = binary variable for build-up of fixed capacity of producer j,
- y_k^D = binary variable for build-up of fixed capacity of distributor k.
- Using the above notations the problem can be formulated as follows:

The model has two objectives. The first one expresses minimizing of total costs. The second one expresses minimizing of total environmental pollution.

$$\text{Min } z_1 = \sum_{j=1}^{n} f_j^P y_j^P + \sum_{k=1}^{p} f_k^D y_k^D + \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} c_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} c_{kl}^D x_{kl}^D$$

$$\text{Min } z_2 = \sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij}^S x_{ij}^S + \sum_{j=1}^{n}\sum_{k=1}^{p} e_{jk}^P x_{jk}^P + \sum_{k=1}^{p}\sum_{l=1}^{r} e_{kl}^D x_{kl}^D$$

Subject to the following constraints:

- the amount sent from the supplier to producers cannot exceed the supplier capacity

$$\sum_{j=1}^{n} x_{ij} \le a_i, \quad i = 1, 2, \ldots, m,$$

- the amount produced by the producer cannot exceed the producer capacity
  $\sum_{k=1}^{p} x_{jk} \le b_j y_j,\ j = 1,\ 2,\ \ldots,\ n,$
- the amount shipped from the distributor should not exceed the distributor capacity
  $\sum_{l=1}^{r} x_{kl} \le w_k y_k,\ k = 1,\ 2,\ \ldots, p,$
- the amount shipped to the customer must equal the customer demand
  $\sum_{k=1}^{p} x_{kl} = d_l,\ l = 1,\ 2,\ \ldots,\ r,$
- the amount shipped out of producers cannot exceed units received from suppliers
  $\sum_{i=1}^{m} x_{ij} - \sum_{k=1}^{p} x_{jk} \ge 0,\ j = 1,\ 2,\ \ldots,\ n,$
- the amount shipped out of distributors cannot exceed quantity received from producers
  $\sum_{j=1}^{n} x_{jk} - \sum_{l=1}^{r} x_{kl} \ge 0,\ k = 1,\ 2,\ \ldots,\ p,$
- binary and non-negativity constraints
  $y_j, y_k \in \{0,1\}, x_{ij}, x_{jk}, x_{kl} \ge 0,\ i = 1,\ 2,\ \ldots,\ m,\ j = 1,\ 2,\ \ldots, n,\ k = 1,\ 2,\ \ldots,\ p,\ l = 1,\ 2,\ \ldots,\ r.$

The formulated model is a multi-objective linear programming problem (MOLP).

3. FROM OPTIMIZING GIVEN SYSTEMS TO DESIGNING OPTIMAL SYSTEMS

Multi-objective linear programming (MOLP) is a model of optimizing a given system by multiple objectives. In MOLP problems it is usually impossible to optimize all objectives together in a given system. Trade-off means that one cannot increase the level of satisfaction for an objective without decreasing this for another objective. Trade-offs are properties of an inadequately designed system and thus can be eliminated through designing better one. The purpose is not to measure and evaluate tradeoffs, but to minimize or even eliminate them. An optimal system should be tradeoff-free. Multi-objective De Novo linear programming (MODNLP) is a problem for designing an optimal system by reshaping the feasible set (Zeleny, 2010).

3.1 Optimizing given systems

Multi-objective linear programming (MOLP) problem can be described as follows.

$$\text{"Max"} \quad z = Cx$$
$$\text{s.t.} \quad Ax \le b, \quad x \ge 0. \tag{1}$$

where C is a (k, n) – matrix of objective coefficients, A is a (m, n) – matrix of structural coefficients, b is an m-vector of known resource restrictions, x is an n-vector of decision variables. In MOLP problems it is usually impossible to optimize all objectives in a given system. For multi-objective programming problems the concept of non-dominated solutions is used (see for example Steuer, 1986). A compromise solution is selected from the set of non-dominated solutions. There are proposed many methods. Most of the methods are based on trade-offs.

3.2 Designing optimal systems

By given prices of resources and the given budget the MOLP problem (1) is reformulated in the MODNLP problem (2)

$$\text{"Max"} \quad z = Cx$$
$$\text{s.t.} \quad Ax - b \le 0,\ pb \le B,\ x \ge 0. \tag{2}$$

where b is an m-vector of unknown resource restrictions, p is an m-vector of resource prices, and B is the given total available budget. From (2) follows $pAx \leq pb \leq B$.

Defining an n-vector of unit costs $v = pA$ we can rewrite the problem (2) as

$$\text{“Max”} \quad z = Cx$$
$$\text{s.t.} \quad vx \leq B \ , \ x \geq 0. \qquad (3)$$

Solving single objective problems

$$\text{Max } z^i = c^i x \quad i = 1,2,\ldots,k$$
$$\text{s.t.} \quad vx \leq B, x \geq 0 \qquad (4)$$

z^* is a k – vector of objective values for the ideal system with respect to B. The problems (4) are continuous “knapsack” problems, the solutions are

$$x_j^i = \begin{cases} 0, j \neq j_i \\ B/v_{j_i}, j = j_i \end{cases} , \qquad \text{where } j_i \in \left\{ j \in (1,\ldots,n) \middle| \max_j (c_j^i / v_j) \right\} .$$

The meta-optimum problem can be formulated as follows

$$\text{Min} \quad f = vx$$
$$\text{s.t.} \quad Cx \geq z^*, x \geq 0. \qquad (5)$$

Solving the problem (5) provides solution: x^*, $B^* = vx^*$, $b^* = Ax^*$. The value B^* identifies the minimum budget to achieve z^* through solutions x^* and b^*. The given budget level $B \leq B^*$. The optimum–path ratio for achieving the best performance for a given budget B is defined as $r_1 = \frac{B}{B^*}$.

The optimum-path ratio provides an effective and fast tool for the efficient optimal redesign of large-scale linear systems. Optimal system design for the budget B: $x = r_1 x^*$, $b = r_1 b^*$, $z = r_1 z^*$.

3.3 De Novo approach for sustainable supply networks

The De Novo approach can be useful in the design of the sustainable supply network. Only a partial relaxation of constraints is adopted. Producer and distributor capacities are relaxed. Unit costs for capacity build-up are computed:

- $p_j^P = \frac{f_j^P}{b_j}$ = cost of unit capacity of potential producer j,
- $p_k^D = \frac{f_k^D}{w_k}$ = cost of unit capacity of potential distributor k.

Variables for build-up capacities are introduced: u_j^P = variable for flexible capacity of producer j, u_k^D = variable for flexible capacity of distributor k.

The constraints for non-exceeding producer and distributor fixed capacities are replaced by the flexible capacity constraints and the budget constraint:

$$\sum_{k=1}^{p} x_{jk} - u_j^P \leq 0, \ j = 1, 2, \ldots, n, \ \sum_{l=1}^{r} x_{kl} - u_k^D \leq 0, \ k = 1, 2, \ldots, p, \ \sum_{j=1}^{n} p_j^P u_j^P + \sum_{k=1}^{p} p_k^D u_k^D \leq B.$$

4. MANAGING SUPPLY NETWORKS BY DOUBLE COMBINATORIAL AUCTIONS

We propose to use a model of double combinatorial auctions as trading model between layers of supply network.

4.1 Double auctions

Auctions with multiple buyers and multiple sellers are becoming increasing popular in electronic commerce. Double auctions are not so often studied in the literature as single-sided auctions (see Xia et al., 2005). For double auctions, the auctioneer is faced with the task of matching up a subset of the buyers with a subset of the sellers. The profit of the auctioneer is the difference between the prices paid by the buyers and the prices paid to the sellers. The objective is to maximize the profit of the auctioneer given the bids made by sellers and buyers.

We present a double auction problem of indivisible items with multiple sellers and multiple buyers. Let us suppose that m potential sellers $S_1, S_2, \ldots, S_m$ offer a set R of r items, $j = 1, 2, \ldots, r$, to n potential buyers $B_1, B_2, \ldots, B_n$. A bid made by seller S_h, $h = 1, 2, \ldots, m$, is defined as $b_h = \{C, c_h(C)\}$, a bid made by buyer B_i, $i = 1, 2, \ldots, n$, is defined as $b_i = \{C, p_i(C)\}$, where

$C \subseteq R$, is a combination of items,
$c_h(C)$, is the offered price by seller S_h for the combination of items C,
$p_i(C)$, is the offered price by buyer B_i for the combination of items C.

Binary variables are introduced for model formulation: $x_i(C)$ is a binary variable specifying if the combination C is assigned to buyer B_i ($x_i(C) = 1$), $y_h(C)$ is a binary variable specifying if the combination C is bought from seller S_h ($y_h(C) = 1$).

$$\sum_{i=1}^{n} \sum_{C \subseteq R} p_i(C)\, x_i(C) - \sum_{h=1}^{m} \sum_{C \subseteq R} c_h(C)\, y_h(C) \rightarrow \max$$

$$\text{s.t.} \quad \sum_{i=1}^{n} \sum_{C \subseteq R} x_i(C) \leqq \sum_{h=1}^{m} \sum_{C \subseteq R} y_h(C), \quad \forall j \in R, \tag{6}$$

$$x_i(C) \in \{0, 1\}, \ \forall\, C \subseteq R, \forall\, i,\ i = 1, 2, \ldots, n, y_h(C) \in \{0, 1\}, \ \forall\ C \subseteq R, \ \forall\, h,\ h = 1, 2, \ldots, m.$$

The objective function expresses the profit of the auctioneer. The constraints ensures for buyers to purchase a required item and that the item must be offered by sellers.

4.2 Solving of double auctions

The formulated combinatorial double auction can be transformed to a combinatorial single-sided auction. Substituting $y_h(C)$, $h = 1, 2, \ldots, m$, with $1 - x_i(C)$, $i = n+1, n+2, \ldots, n+m$, and substituting $c_h(C)$, $h = 1, 2, \ldots, m$, with $p_i(C)$, $i = n+1, n+2, \ldots, n+m$, we get a model of a combinatorial single-sided auction.

$$\sum_{i=1}^{n+m} \sum_{C \subseteq R} p_i(C)\, x_i(C) - \sum_{i=n+1}^{n+m} \sum_{C \subseteq R} p_i(C) \rightarrow \max$$

$$\text{s.t.} \quad \sum_{i=1}^{n+m} \sum_{C \subseteq R} x_i(C) \leqq m, \ \forall j \in R, \tag{7}$$

$$x_i(C) \in \{0, 1\}, \ \forall\ C \subseteq R, \ \forall\, i,\ i = 1, 2, \ldots, n+m.$$

The model (7) can be solved by methods for single-sided combinatorial auctions. Complexity is a fundamental question in combinatorial auction design. The algorithms proposed for solving combinatorial auctions are exact algorithms and approximate ones. Many researchers consider iterative auctions as an alternative.

One way of reducing some of the computational burden in solving the problem is to set up a fictitious market that will determine an allocation and prices in a decentralized way. In the iterative approach, there are multiple rounds of bidding and allocation and the problem is solved in an iterative and incremental way. Iterative combinatorial auctions are attractive to bidders because they learn about their rivals' valuations through the bidding process, which could help them to adjust their own bids (see Parkes, 2001). The key challenge in the iterative combinatorial auctions design is to provide information feedback to the bidders (see Pikovsky and Bichler, 2005). We propose to use an iterative approach for combinatorial auctions. The primal-dual approach is used for solving. For the problem (7) we will formulate the LP relaxation and its dual. The scheme can be outlined as follows:

1. Choose initial prices for sellers and buyers.
2. Announce current prices and collect bids.
3. Compute the current dual solution by interpreting the prices as dual variables. Try to find a feasible allocation, an integer primal solution that satisfies the stopping rule. If such solution is found, stop and use it as the final allocation. Otherwise update prices and go back to 2.

5. CONCLUSION

De Novo approach was applied for sustainable supply network design problem and provides better solution than traditional approaches applied on fixed constraints. The design problem was formulated as MOLP problem. The economic and environmental objectives were used in the model but multiple objectives can be used in general. De Novo programming (DNP) approach is open for further extensions as fuzzy DNP, interval DNP, complex types of objective functions and continuous innovations. The numerous applications in electronic commerce have led to a great deal of interest in double auctions. The paper presents a model of combinatorial double auctions for managing supply networks. The formulated model can be transformed to a combinatorial single-sided auction and solved by methods for single-sided combinatorial auctions. We propose to use an iterative approach to solving combinatorial double auctions. The primal-dual algorithm can be taken as a decentralized and dynamic method to determine equilibrium.

ACKNOWLEDGMENT

The research project was supported by the grant No. 13-07350S of the Grant Agency of the Czech Republic and by Grant No. IGA F4/54/2015, Faculty of Informatics and Statistics, University of Economics, Prague.

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