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LOCATION ROUTING MODEL FOR DESIGNING PLASTICS RECYCLING NETWORK

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Abstract: *This paper proposes location-routing model for designing recycling network with profit. Proposed model simultaneously determine collection points' locations with distance-dependent returns, location of intermediate consolidation points (transfer centers) and the route of the collection vehicle so as to maximize its profit from the collection of recyclables. Routing part of the model was formulated as a multiple matching problem.*

Keywords*: location-routing problem, recycling network, MILP*

1. INTRODUCTION

The location-routing problems (LRP) merges facility location and vehicle routing into a single problem where strategic location and tactical/operational routing decisions are taken simultaneously. This integrated approach has found to be useful in several real-life applications (Nagy and Salhi, 2007). In this paper we consider a LRP in plastics recycling. Namely, it is estimated that 66.5 million tons of plastic will be placed on the European Union (EU) market in 2020 and global plastic production could triple by 2050 (Green Paper, 2013). Once in the environment, plastic waste can persist for hundreds of years (Green Paper, 2013). In order to deal with the problem of plastic waste, EU introduced legislation like the Packaging Directive 94/62/EC and Framework Directive on waste 2008/98/EC. The Packaging Directive has a specific recycling target for plastic packaging, while the Framework Directive on waste sets a general recycling target for household waste which covers plastic waste (Green Paper, 2013). For achieving imposed recycling targets, it is necessary to establish appropriate logistics network structures. This logistics networks must be convenient for end users (González-Torre and Adenso-Díaz, 2005), since the participation of end users is crucial for a successful achieving any recovery target set by legislation, because they are responsible for separation of these products at their residence and carrying them to designated collection points (CPs). Although, efficient source segregation collection can contribute significantly to maximizing material recycling, but can represent up to 70% of the entire cost of waste management (Dogan and Duleyman, 2003).

From here, the main intention of this paper is to propose a model for designing recycling logistics network (RN) with profit. The LR model for designing RN with profit proposed here, has the following specificities. Location part of the model includes decisions of the positioning both CPs as a lower level of the network and transfer stations (TSs) at the higher level of the network. The revenue obtained from quantity of recyclables for specific CP is related with the

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proximity of the CP to the end users. So, we introduced distance dependent quantity of recyclables dropped off to CPs introducing collection rate as a function of distance. Particularly, we consider a problem where end users are located in city blocks. Most of the today modern cities have so called block structures, characterized with buildings in which residents live, and internal streets network within. In order to collect recyclables in such environment, CPs must be located along these internal streets. Therefore, routing part of the model gives opportunity for considering whole blocks as a network nodes to be visited by collection vehicles. Internal street network in city blocks is usually simple, so it is possible to determine optimal route through blocks in advance. Another specificity of the proposed approach is that while performing one collection route, due to its limited capacity, vehicle can visit only small number of city blocks. This facts gives opportunity to formulate and solve routing part of the problem as multiple matching problem.

The remainder of the paper is organized as follows. Section 2 presents the problem description, and mathematical formulation of the described problem. Section 3 gives numerical example. Finally, section 4 concludes the study.

2. MATHEMATICAL FORMULATION

Most of the today modern cities have so called block structures. These blocks can be in a variety of sizes and shapes, and they are characterized with buildings in which residents live and road network within. In this paper, term end user refers to a building inside the specific city block (we aggregated residents to its residential building considering them as a single end user). Each end user is characterized by the volume of waste produced per day which corresponds to the total quantity generated in all households residing in a building. Potential locations of CP are characterized by its capacity and distances to all end users in each city block. In order to model the influence of distance between end users and CPs on the collecting of recyclables, we assume that recyclables collection rate $f_d \in [0,1]$ is a known function of distance. This function models the influence of distance between end users and CPs, in way that collection rate is inversely proportional to distance (Berman et al., 2003). We define two characteristic distances *l* and *u* (l < u), between the end user and CP where *l* represents the lower and *u* upper bound of walking distance to CP for each end user. When the distance *d* from the end user to the closest CP is $0 \le d \le l$ then *f(d)=1*, while in case when $d \ge u$, *f(d)=0*. If the distance between the end user and CP is $l \le d \le u$, we assumed that the collection rate corresponds to $f(d) = \frac{u-d}{u-l}$. In the routing part of the problem, there is a capacitated vehicle that has to visit CPs located in city blocks, originating from TS. Transfer station represents a facility in which recyclables are inspected and consolidated for further processing. We have a set of potential locations for TSs, characterized by costs of opening TSs. The length of inner streets in city blocks may differ depending of the city block shape and size, but once the vehicle enters the city block it could only be traverse through these inner streets, along which CPs are located. The length of the route through city block is always the same, regardless of the number of stops per CP. This fact enables us to route distance only from TS to city blocks, while route part when vehicle traverse city block is predetermined and included in routing costs. More importantly this fact, gives us opportunity to formulate vehicle routing part of the LR problem as a multiple matching problem instead of classical VRP formulations. Also, we include idling time at each CP in costs calculation. Assumptions of the proposed LR model are:

- TS is assumed to be uncapacitated
- The route in city block can be predefined as the CPs are placed along the internal streets within the city block, whose network is short and simple, and the optimal internal route within the block can easily be determined by solving arc routing problem. The internal

route has constant length which is passed always when block is visited and does not depends on the number of CPs opened

- Quantities of recyclables that are generated during the observation period in city block do not allow the vehicle to serve more than four blocks in one route
- In order to propose a MILP model for the problem the following notation is introduced.

Sets:

$I=\{1,..i..\lvert I\rvert\}$	set of end users
$B=\{1,..b..\lvert B\rvert\}$	set of city blocks
$K=\{1,...k...\lvert K\rvert\}$	set of potential sites for CPs
$J=\{1,...j...\lvert J\rvert\}$	set of potential sites for transfer points

Parameters:

R	revenue from selling collected quantity of recyclables
d_{ikb}	walking distance between end user i to k-th CP in a city block b
Z_{ikb}	collection rate for the distance d_{ikb}, $Z_{ikb}=\begin{cases}1, & when\ 0\le d_{ikb}\le l\\ f(d_{ikb}), & when\ l<d_{ikb}\le u\\ 0, & when\ d_{ikb}>u\end{cases}$
F_k	costs of opening CPs $k\in K$
F_j	costs of opening transfer points $j\in J$
α_{kb}	idling time costs at CP $k\in K$ in city block $b\in B$
Q_r	capacity of vehicle (or route)
Q_j	capacity of transfer station $j\in J$
Q_{kb}	capacity of CP $k\in K$ in city block $b\in B$
Q_{ib}	available quantity of recyclables at end user $i\in I$ in city block $b\in B$

C_{jpqwe}, C_{jpqw}, C_{jpq}, C_{jp} costs of visiting CPs in blocks $p,q,w,e\in B$ in a single route (including costs from/to transfer station $j\in J$ and costs inside city blocks), respectively for routes visiting four, three, two, and one city block $b\in B$. Cost of routes through city blocks are added to these cost.

Big M number (sufficiently large number)

Decision variables:

$$Y_{kb}=\begin{cases}1,\ if\ collection\ point\ k\in K\ is\ opened\ in\ city\ block\ b\in B\\ 0, otherwis\ e\end{cases}$$

$$Y_{j}=\begin{cases}1,\ if\ transfer\ station\ j\in J\ is\ opened\\ 0, otherwis\ e\end{cases}$$

$$Y_{jpqwe}=\begin{cases}1,\ if\ nodes\ p,q,w,e\in B\ are\ merged\ in\ the\ same\ route\ from\ transfer\ point\ j\in J\\ 0, otherwis\ e\end{cases}$$

$$Y_{jpqw}=\begin{cases}1,\ if\ nodes\ p,q,w\in B\ are\ merged\ in\ the\ same\ route\ from\ transfer\ point\ j\in J\\ 0, otherwis\ e\end{cases}$$

$$Y_{jpq}=\begin{cases}1,\ if\ nodes\ p,q,\in B\ are\ merged\ in\ the\ same\ route\ from\ transfer\ point\ j\in J\\ 0, otherwis\ e\end{cases}$$

$$Y_{jp} = \begin{cases} 1, \text{ if node } p, \in B \text{ are served in the direct route from transfer point } j \in J \\ 0, \text{otherwis e} \end{cases}$$

$X_{ikb} \leq 1$ defines fraction of recyclables brought from end user to $i \in I$ collection site $k \in K$ in city block $b \in B$.

The mathematical formulation of the proposed MILP model is given below:

$$Max \quad \sum_b \sum_k \sum_i RQ_{ib} X_{ikb} - \sum_b \sum_k (F_{kb} + \alpha_{kb}) Y_{kb} - \sum_j F_j Y_j - \sum_j \sum_{p \in B} \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} \sum_{e \in B/(\{p\} \cup \{q\} \cup \{w\})} C_{jpqwe} Y_{jpqwe} - \sum_j \sum_{p \in B} \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} C_{jpqw} Y_{jpqw} - \sum_j \sum_{p \in B} \sum_{q \in B/\{p\}} C_{jpq} Y_{jpq} - \sum_j \sum_{p \in B} C_{jp} Y_{jp} \tag{1}$$

s.t.

$$Q_{ib} X_{ikb} \leq Z_{ikb}, \quad \forall\, i, k, b \tag{2}$$

$$\sum_k X_{ikb} \leq 1 \quad \forall, i, b \tag{3}$$

$$\sum_i Q_{ib} X_{ikb} \leq Y_{kb} Q_{kb} \quad \forall k, b \tag{4}$$

$$X_{ikb} \leq Y_{kb} \quad \forall k, b \tag{5}$$

$$\sum_j \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} \sum_{e \in B/(\{p\} \cup \{q\} \cup \{w\})} Y_{jpqwe} + \sum_j \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} \sum_{e \in B/(\{p\} \cup \{q\} \cup \{w\})} Y_{jqpwe} + \sum_j \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} \sum_{e \in B/(\{p\} \cup \{q\} \cup \{w\})} Y_{jqwpe} + \sum_j \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} \sum_{e \in B/(\{p\} \cup \{q\} \cup \{w\})} Y_{jqwep} + \sum_j \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} Y_{jpqw} + \sum_j \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} Y_{jqpw} + \sum_j \sum_{q \in B/\{p\}} \sum_{w \in B/(\{p\} \cup \{q\})} Y_{jqwp} + \sum_j \sum_{q \in B/\{p\}} Y_{jpq} + \sum_j \sum_{q \in B/\{p\}} Y_{jqp} + Y_{jp} = 1, \quad \forall p \tag{6}$$

$$Y_{jpqwe} \leq Y_j, \forall j \in J, \forall p \in B, \forall q \in B/\{p\}, \forall w \in B/(\{p\} \cup \{q\}), e \in B/(\{p\} \cup \{q\} \cup \{w\}) \tag{7}$$

$$Y_{jpqw} \leq Y_j, \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}) \tag{8}$$

$$Y_{jpq} \leq Y_j, \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\} \tag{9}$$

$$Y_{jp} \leq Y_j, \forall j \in J, \quad \forall p \in B \tag{10}$$

$$\sum_i \sum_k Q_{ip} X_{ikp} + \sum_i \sum_k Q_{iq} X_{ikq} + \sum_i \sum_k Q_{iw} X_{ikw} + \sum_i \sum_k Q_{ie} X_{ike} - M(1 - Y_{jpqwe}) \leq Q_r Y_{jpqwe}, \\ \forall j \in J; \quad \forall p \in B; \quad \forall q \in B/\{p\}; \quad \forall w \in B/(\{p\} \cup \{q\}); \quad e \in B/(\{p\} \cup \{q\} \cup \{w\}) \tag{11}$$

$$\sum_i \sum_k Q_{ip} X_{ikp} + \sum_i \sum_k Q_{iq} X_{ikq} + \sum_i \sum_k Q_{iw} X_{ikw} - M(1 - Y_{jpqw}) \leq Q_r Y_{jpqw}, \\ \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}) \tag{12}$$

$$\sum_i \sum_k Q_{ip} X_{ikp} + \sum_i \sum_k Q_{iq} X_{ikq} - M(1 - Y_{jpq}) \leq Q_r Y_{jpq}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\} \tag{13}$$

$$\sum_i \sum_k Q_{ip} X_{ikp} - M(1 - Y_{jp}) \leq Q_r Y_{jp}, \quad \forall j \in J, \quad \forall p \in B \tag{14}$$

$$\sum_k Y_{kp} \geq Y_{jpqwe}, \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}), \quad e \in B/(\{p\} \cup \{q\} \cup \{w\}) \tag{15}$$

$$\sum_k Y_{kq} \geq Y_{jpqwe}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}), \quad e \in B/(\{p\} \cup \{q\} \cup \{w\}) \tag{16}$$

$$\sum_k Y_{kw} \geq Y_{jpqwe}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}), \quad e \in B/(\{p\} \cup \{q\} \cup \{w\}) \tag{17}$$

$$\sum_k Y_{ke} \geq Y_{jpqwe}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}), \quad e \in B/(\{p\} \cup \{q\} \cup \{w\}) \tag{18}$$

$$\sum_k Y_{kp} \geq Y_{jpqw}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}) \tag{19}$$

$$\sum_k Y_{kq} \geq Y_{jpqw}, \quad \forall j \in J \quad \forall p \in B \quad \forall q \in B/\{p\} \quad \forall w \in B/(\{p\} \cup \{q\}) \tag{20}$$

$$\sum_k Y_{kw} \geq Y_{jpqw}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\}, \quad \forall w \in B/(\{p\} \cup \{q\}) \tag{21}$$

$$\sum_k Y_{kp} \geq Y_{jpq}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\} \tag{22}$$

$$\sum_k Y_{kq} \geq Y_{jpq}, \quad \forall j \in J, \quad \forall p \in B, \quad \forall q \in B/\{p\} \tag{23}$$

$$\sum_k Y_{kp} \geq Y_{jp}, \quad \forall j \in J, \quad \forall p \in B \tag{24}$$

$$Y_{kb} \in \{0,1\}, Y_j \in \{0,1\}, Y_{jpqwe} \in \{0,1\}, Y_{jqpwe} \in \{0,1\}, Y_{jqwpe} \in \{0,1\}, Y_{jqwep} \in \{0,1\}, Y_{jpqw} \in \{0,1\}, Y_{jqpw} \in \{0,1\}, Y_{jqwp} \in \{0,1\},$$
$$Y_{jpq} \in \{0,1\}, Y_{jpq} \in \{0,1\}, Y_{jp} \in \{0,1\}, X_{ikb} \leq 1, X_{ikb} \geq 0 \tag{26}$$

The objective function maximizes the profit which is composed of obtained revenue from the collected recyclables (term 1 in objective function), minus the fixed cost of opening CPs and idling time costs in each city block (term 2), costs of locating transfer points (term 3), an routing costs (terms 4, 5, 6 and 7). First constraint set (2) represents the quantity of recyclables to be collected under ''coverage decay function''. Constraints (3) ensures that fraction of recyclables assigned to CP *k* is less or equal to quantity generated at end user *i*. Quantity of recyclables assigned to CP cannot exceed the capacity of CP (4), while constraints (5) ensures that recyclables are assigned to only opened CPs. Constraints (6) prohibit multiple visits of the same node (city block b), and provide that each city block must be visited exactly once. Constraint sets (7)-(10) ensures that vehicles starts tour only from transfer point which is opened. Constraint sets (11)-(14) ensures that vehicle capacity isn't exceeded. Constraint sets (15)-(24) ensures that only city blocks in which CPs are opened are visited in the route, Finally, constraint sets (26) define nature of the variables.

3. NUMERICAL EXAMPLE

To test proposed MILP model for designing RN with profit, we generated three different problem instance sets: small (S), medium (M) and large (L) instances. The main difference between instances is in number of end users and spatial distribution in city blocks. For all set of problems we generated 240 instances (Table 1). The execution of the instances (CPU time) has been limited to 7200 seconds. Models have been solved with the usage of Cplex 12.6, and instances were run on an Intel(R) Core(TM) i5-4200U (2.30 GHz, RAM 8Gb).

Table 1. Input parameters of the model

Input parameters	S	M	L
F_k(€/day)	0.5		
F_j(€/day)	0		
Costs per stop (€/per stop)	0.002		
revenue(€/kg)	0.12264		
Q_k(kg)	20		
Q_r(kg)	200	300	800
l(m)	50		
u(m)	400		
Time horizon	2.5 day		
No. of city blocks	{4,5,6,8,10,12,15,20}		
No. of potential location for transfer points	2		
Number of residents in each city block	Beta (2,5) distribution in the range[48,200]		
Generated quantity of recyclables in (Q_{ib})	Uniform distribution (0.8;0.1)* P (P-% of plastics in municipal waste)		
Number of potential CPs	(Q_{ib}/Q_k)+1		
Distances between city blocks (m)	Randomly generated in the range [100,700]		
Distances between transfer points and city blocks (m)	Randomly generated in the range [1000,10000]		
Distances inside city blocks (m)	Randomly generated in range[400,1200]		
Distances between end users and potential locations for CPs	Beta (2,5) distribution in the range[15,400]		
Cost per km traveled (€/km)	0.0006 (inside the block x3)		
Number of end users in each city block (Randomly generated in the shown range :)	[4,7]	[5,25]	[10,70]

Due to the complexity of proposed LR model, the instances that can be solved to optimality are typically of small size. When solving large and medium instances in preliminary tests, computational difficulties were faced due to the hard combinatorial problem characteristics. For medium instances, instances up to 8 city block (not all of it) could be solved to optimality. In case of large instances, even the smallest number of city blocks caused difficulties, while the biggest number of city blocks could not be loaded into computer memory (Table 2). Although this results present the beginning of the research in this area, it is clear that proposed MILP model needs development of appropriate heuristics or application of metaheuristics for solving models of larger size.

Table 2. Results of the proposed LR model

No. of city blocks	*S instances*			*M instances*			*L instances*		
	*#opt***	*Fopt**	*CPU-opt(s)**	*#opt***	*Fopt**	*CPU-opt(s)**	*#opt***	*Fopt**	*CPU-opt(s)**
4	10/10	2.04	12.59	9/10	24.32	313.81	1/10	134.37	11.80
5	10/10	-0.26	12.16	9/10	40.37	104.20	0/10	/	/
6	10/10	-0.05	0.83	3/10	47.85	20.67	0/10	/	/
8	9/10	5.95	55.98	1/10	53.3	98.83	0/10	/	/
10	10/10	3.40	25.33	0/10	/	/	0/10	/	/
12	10/10	7.244803	67.42	0/10	/	/	0/10	/	/
15	10/10	9.268272	632.88	0/10	/	/	0/10	/	/
20	10/10	15.56321	537.26	0/10	/	/	a*	a*	a*

*average value; ** time limit 7200s;a* couldn't be loaded into the memory

3. CONCLUSION

This paper presents a possible approach to define the optimal network for recycling plastic waste, by presenting LR formulation for this problem. Preliminary testing of the proposed approaches is promising, but numerous aspects of the problem and application of approach proposed need future research. Future research direction may include examining the system behavior with different system parameters (different vehicle capacity, different capacity of the containers, etc). But most importantly, development of appropriate heuristics for solving models of larger size.

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