OPTIMIZATION OF VEHICLE ROUTING DEPENDING ON VEHICLE LOAD

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Abstract: *Nowadays many subjects pay attention to reduction of environmental externalities of greenhouse gases emissions. Unsurprisingly the transport is one of the areas significantly contributing to the production of CO_2 emissions. One of important areas in the transport logistic is analyzing various routing problems simultaneously with using integrated optimization approaches where one of the source of emissions reduction are shorter routes, and better management of vehicle loads. Obviously, greenhouse emissions are directly reflected by fuel consumption. The paper is focused on modified vehicle routing problem that enables setting vehicle routes depending on vehicle load. The illustrative example presenting difference compared to classical vehicle routing problem is also given.*

Keywords: *capacitated vehicle routing problem*

INTRODUCTION

A variety of optimization models to support decision making of distribution companies are commonly known. Distribution companies often implement models aimed at minimizing the total traveled distance. However, when analyzing the fuel consumption, it is clearly observed that the traveled distance is not the only relevant factor. Undoubtedly also vehicle load has significant impact on the fuel consumption. The paper is aimed on model that minimizes the fuel consumption, depending on the length of the traveled route and also on the vehicle load. The paper is structured as follows: In the first section we present a classic model of distribution [(Čičková, Brezina, Pekár 2013), (Federgruen, Simchi-Levi 1995), (Golden, Raghavan, Wasil 2008), (Miller, Tucker, Zemlin 1960)] which serves as the basis for constructing a modified model involving the vehicle load. The modified model is given in the second part. The third part is devoted to an illustrative example, where we calculated optimum route based on classical model as well as on a modified model. The results illustrate the difference using different approaches, while we also report an achieved decrease in the fuel consumption.

1. CAPACITATED VEHICLE ROUTING PROBLEM

This section is devoted to the capacitated vehicle routing problem (CVRP). The idea of this model is further refined in part 2. The formulation CVRP is based on the graph representation. Consider a graph with n + 1 nodes. Let $N_0 = \{0,1,...,n\}$ be the set of nodes representing the location of customers as well as the depot (origin). The standard CVRP consists in designing the optimal set of routes for a vehicle (vehicles) in order to serve a given set of customers located in a certain nodes of the net representing by subset $N=\{1,2,...,n\}$. Each customer has a certain demand (g_i, $i\in N$). The demand need to be met from origin ($\{0\}$). Further on there exists a matrix $\mathbf{D}_{(n+1)\text{x}(n+1)}$ that represents the minimum distances between all the pairs of nodes (customers and the depot). The optimal vehicle routes must be designed in such a way that each customer is visited only once by exactly one vehicle, all routes start and end at the origin, and the total demands of all customers on one particular route must not exceed the capacity of the vehicle (g). Consider that demands of customers are met in full and also consider the individual demands do not exceed the capacity of the vehicle, otherwise a quantity equal to the vehicle capacity is realized in self-route and the model includes only the remaining demand.

Based on this assumption, the model can be stated as follows:

$$\min f(\mathbf{X},\mathbf{u})=\sum_{i\in N_0}\sum_{\substack{j\in N_0\\ i\neq j}} d_{ij}x_{ij}+k\sum_{i\in N}u_i \tag{1}$$

$$\sum_{i\in N_0} x_{ij}=1,\ j\in N\ ,\ i\neq j \tag{2}$$

$$\sum_{j\in N_0} x_{ij}=1\ ,\ i\in N\ ,\ i\neq j \tag{3}$$

$$u_i+q_j-g(1-x_{ij})\leq u_j\ ,\ i\in N_0\ ,\ j\in N\ ,\ i\neq j \tag{4}$$

$$q_i\leq u_i\leq g\ ,\ i\in N \tag{5}$$

$$u_0=0 \tag{6}$$

$$x_{ij}\in\{0,\ 1\}\ ,\ i,j\in N_0\ ,\ i\neq j \tag{7}$$

Mathematical programming formulation of CVRP requires two types of variables: the binary variables x_{ij}, $i,j\in N_0$ where x_{ij} = 1 if the edge between node i and node j is used, otherwise x_{ij} = 0 (7). Further on the free variables u_i, $i\in N$ based on well-known Miller-Tucker-Zemlin's formulation of the traveling salesman problem (Miller, Tucker and Zemlin, 1960) are used. These variables enable to calculate the load of vehicle.

Objective function (1) models the total traveled distance. Adding the expression $k\sum_{i\in N}u_i$ (k is small positive number) enables setting the exact vehicle load to i-th, $i\in N$ node (including). To obtain the real value of traveled distance, if necessary, we can subtract the value of before mentioned expression. Equations (2) and (3) ensure that each customer (except the origin) is visited exactly ones. Equations (4) are anti-cyclical conditions that prevent the formation of such sub-cycles which do not contain an origin. The set of variables u_i, $i\in N$ also ensures the calculation of current load of vehicles in its route to i-th customer (including) and together with equations (5) they ensure that the current load does not exceed the capacity of vehicle, which is set to zero (6) in the origin.

2. CAPACITATED VEHICLE ROUTING PROBLEM DEPENDING ON VEHICLE LOAD

Consider the same preconditions as in part 1, but besides to existing known parameters (inputs to CVRP) also consider new known parameters associated with the fuel consumption. Let parameter c_0 be the vehicle consumption per unit distance and let c_1 be parameter representing the increase in consumption per unit distance for 1 unit of vehicle load. The goal of capacitated vehicle routing problem depending on vehicle load (CVRPVL) is to establish such distribution, which minimizes whole vehicle's fuel consumption (not minimizing the distance) under the same conditions as in CVRP (2) – (7). The objective function (1) is replaced by function (8), Its meaning is as follows: minimizing the expression $\sum_{i\in N_0}\sum_{\substack{j\in N_0\\ i\neq j}} c_0 d_{ij} x_{ij}$ enables modeling of the fuel consumption depending on the traveled route. While variables u_i, $i\in N$ and u_0 = 0 in model (1)-(7) enable setting the vehicle load, increased consumption depending on the load can be modeled by expression $\sum_{i\in N_0}\sum_{\substack{j\in N_0\\ i\neq j}} c_1 \cdot u_i d_{ij} x_{ij}$.

Model CVRPVL can be stated as follows:

$$\min f(\mathbf{X},\mathbf{u}) = \sum_{i\in N_0}\sum_{\substack{j\in N_0\\ i\neq j}} (c_0 + c_1 \cdot u_i) d_{ij} x_{ij} \quad (8)$$

$$\sum_{i\in N_0} x_{ij} = 1,\ j\in N,\ i\neq j \quad (9)$$

$$\sum_{j\in N_0} x_{ij} = 1,\ i\in N,\ i\neq j \quad (10)$$

$$u_i + q_j - g(1-x_{ij}) \le u_j,\ i\in N_0,\ j\in N,\ i\neq j \quad (11)$$

$$q_i \le u_i \le g,\ i\in N \quad (12)$$

$$u_0 = 0 \quad (13)$$

$$x_{ij} \in \{0,\ 1\},\ i, j\in N_0,\ i\neq j \quad (14)$$

It should be noted that the resulting routes depend on the type of the problem. When goods are collected than the optimum route represented by variables x_{ij} = 1, $i, j\in N_0$ is the route from *i*-th node to *j*-th node. In the case of distribution those variables represent the route from *j*-th to *i*-th node. In the next section we present an illustrative example of distribution of goods.

3. ILLUSTRATIVE EXAMPLE

Let us introduce the problem dealing with a distribution scheduling. The problem can be described by network consisting of origin from where 7 nodes need to be served, so that $N=\{1,2,...,7\}$, $N_0=\{0\}\cup N$. Known customers' demands are elements of 7-dimensional vectors **q**, **q** = (5, 2, 10, 10, 6, 8, 9)^T. Further on suppose the vehicle consumption per unit distance (c_0) is set to 1 and also suppose there is 3 percent increase in consumption per distance unit and per unit of load (c_0 = 0.03).

Table 1. Matrix of Minimal Distances between all the Pairs of Nodes

	0	1	2	3	4	5	6	7
0	0	200	400	100	390	100	50	170
1	200	0	200	120	200	150	170	100
2	400	200	0	300	40	300	350	230
3	100	120	300	0	300	90	50	140
4	390	200	40	300	0	290	350	220
5	100	150	300	90	290	0	80	70
6	50	170	350	50	350	80	0	150
7	170	100	230	140	220	70	150	0

The daily distribution need to be provided by vehicles with the same capacity (g = 24). The number of vehicles can be adapted as required; therefore it is not necessary to consider limits on the initial number of vehicles. The known matrix of shortest distances **D** between all nodes is given in Table 1.

Model CVRP (1) – (7) and model (8) – (14) were implemented in software GAMS (solver Cplex (CVRP), Couenne (CVRPVL)).

Firstly, model (1) – (7) was implemented. Results are given in Table 2.

Table 2 **CVRP Route**

Route	Sequence	Distance	Fuel consumption depending on load
Route	0-1-2-4-5-0-6-0-7-3-0	1340	1764.20
Route 1	0-1-2-4-5-0	830	1147.40
Route 2	0-6-0	100	112.00
Route 3	0-7-3-0	410	504.80

Source: Own compilation.

Then, model (8) – (14) was implemented. Results are given in Table 3.

Table 3 **CVRPVL Route**

Route	Sequence	Distance	Fuel consumption depending on load
Route	0-1-2-4-0-6-3-0-5-7-0	1370	1661.90
Route 1	0-1-2-4-0	830	1016.00
Route 2	0-6-3-0	200	242.00
Route 3	0-5-7-0	340	403.90

Source: Own compilation.

The change in resulting route is observed, when we provide two different objectives (1) and (8). In the case of implementing model (1) - (7) we obtain the value of the total traveled distance 1340. Using model (8)–(14) leads to route length of 1370. But when we calculate the fuel consumption depending on vehicle load, resulting values are those: 1764.20 in the case of model (1)–(7) and 1661.90 in the case of model (8)–(14). Using CVRPVL model enables decreasing fuel consumption of 5.8 percent. Changes are caused by routes structure when rearrange nodes following the goal of minimizing the consumption according to the vehicle load.

3. CONCLUSION

Classic routing problem take into account only one factor affecting the cost of delivery, which is the traveled distance. When realizing real distribution the costs are affected by several factors: traveled distance, the weight of goods, route gradient, weather situation, etc. The authors constructed a model that enables taking into account not only traveled distance but also the weight of the loaded goods.

The first part of the paper is devoted to classic model of distribution, while the second part is aimed at its modification based on the increase in consumption depending on the weight of loaded goods. The next section gives an example of using mentioned models, while justifying the difference in the results obtained. Also the results of illustrative example show that when solving practical problems it is necessary to include the weight of the load, since that parameter largely affects the individual routes. Using of CVRPVL model enables to achieve reduction in fuel consumption by 5.8 percent.

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