

## A MULTI-OBJECTIVE MODEL FOR UNDESIRABLE FACILITY LOCATION

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**Abstract:** *This paper presents a bi-objective mixed integer mathematical model for siting landfills. The first objective minimizes total cost of facility establishing and entire demand satisfaction while the second objective minimizes total number of end users undesirably influenced by landfills. The model is tested on small scale illustrative example and all Pareto optimal solutions are obtained. The solutions are presented and discussed.*

**Keywords:** *landfill location, bi-objective mixed integer modeling, Pareto optimal solutions.*

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### 1. INTRODUCTION

Landfilling represents the oldest form of municipal solid waste management and the least desirable option according to waste treatment hierarchy. However, only in the EU in 2010 total waste production was around 2.5 billion tons of which 36% was recycled, while the rest was landfilled or burned (EC, 2017). Hence, the landfilling of waste unfortunately is still dominant waste treatment option even in the countries with strict environmental laws.

Landfills are facilities that pose environmental risks and fall in the category of undesirable facilities. In most practical problems locating undesirable facilities involves multiple objectives that are often conflicted. In the case of landfill locations, for example, most people want landfill to be located as far as possible from population centers. However, landfill locations tend to be close to highly populated areas as important waste generation sites to minimize transportation costs. Additionally, landfills pose serious environmental risk due to pollution of the local environment manifested in contamination of groundwater or soil, generation of dangerous gases, air pollution, reduced local property values, etc.

The last two decades have seen many new multi-objective models and approaches in landfill selection. Caruso et al. (1993) presented multi-objective model, in which three different objectives (the overall cost, the waste of recyclable resources, and the environmental impact) were taken into account. The three objectives were then combined into a parametric single objective according to the weighting method. A set of approximate Pareto solutions was searched through an add-drop heuristic. Rahman and Kuby (1995) proposed a multi-objective model locating transfer stations, where the focus was the compromise between minimizing transshipment costs and maximizing the distance of the facilities from the residential zones. They tested their model on actual data from Phoenix, Arizona. Nozik (2001) developed fixed-charge location problem (FCLP) with coverage restrictions. In this paper the objective was to minimize cost while maintaining an appropriate level of service for determining facility

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locations. Two heuristics based on Lagrangian relaxation were used and tested on real data collected from networks in the USA. Rakas et al. (2004) developed a bi-objective model for landfill location proposed to appropriately address uncertainty associated with this class of location problems. The methodology developed in this study is tested using the real-world data from a county in USA. Erkut et al. (2008) examined solid waste management and measured a decreasing function of distance from facilities. A multi-objective MIP model was developed for the location–allocation municipal problem at the regional level. The multi-objective problem was formulated as a lexicographic minimax problem aiming at finding a non-dominated solution with all normalized objectives as close as possible. Interesting multi-criteria approach was introduced by Eiselt and Marianov (2014). The authors examined where to locate landfills and transfer stations in the network, and formulated problem as a bi-objective mixed integer optimization problem. They proposed two objectives, the first one, to minimize costs, as usual, while the second to minimize pollution. The model was tested on real data collected from a region of Chile. Ghiani et al. (2014) provided great review paper about operations research applications in solid waste management. They focused on strategic and tactical issues, and covered more than 65 papers.

In this paper we focused on the model for locating landfills which is based on two objectives. The first objective function, as in the classical FCLP (Balinski, 1964), aims to minimize total facility and transportation costs. The second objective function, inspired by Minimum Covering Location Problem with Distance Constraint (MCLPDC) introduced by Oded and Rongbing (2008), minimizes the total number of end users undesirably influenced by landfills.

The paper is structured as follows. Description of the problem as well as mathematical formulation is represented in Section 2. In Section 3 numerical results are presented, while Section 4 summarizes our findings and provides some thoughts regarding future research.

## 2. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

Even if most people want landfills to be located as far as possible from their communities, the tendency is to locate them closer to highly populated areas as important waste generation sites to minimize transportation costs which is one of the usual objectives when locating landfills. Also, minimizing the number of residents affected by negative impacts of landfills represents another objective when it comes to choosing appropriate landfill site. Because landfills pose serious environmental risk, when locating them, a restriction is imposed that no two selected landfill sites are within a specified distance from each other.

We considered a problem, where end users  $i$  are represented by population centers located at known sites. Typically, population is aggregated at these centers, and we assume  $v_i$  residents are located at site  $i$ , each one generating  $q_i$  kg of waste per day. That means that  $q_i v_i$  kg of waste will be generated at site  $i$ . All generated waste from end users should be collected and transported to the closest landfill. In order to formulate mathematical model, we defined two radiuses, where  $R_1$  represents minimal acceptable separation distance between any two located facilities and  $R_2$  represents separation distance between any landfill location and end users (Figure 1). We are locating landfills so as to minimize overall costs (landfill establishing and entire demand satisfaction) while keeping them on a certain predefined distance from each other and in the same time to minimize their impact on end users.

Following notation is used for mathematical formulation of the described problem.

*Sets and parameters:*

$N$  - discrete set of nodes representing potential landfill sites and end users

$J$  - set of nodes representing potential landfill locations

$I$  - set of nodes representing end users

$$N=I \cup J$$

$j, k$  - indices used to represent potential landfill locations

$i$  - index used to represent end user

$d_{ij}$  - the shortest distance between end user  $i$  and potential location  $j$

$f_j$  - fixed cost of locating a landfill at potential location  $j$

$v_i$  - number of residents located at site  $i$

$q_i$  - quantity of waste generated per person at end user  $i$

$w_i = q_i v_i$  - demand of end users  $i$

$c_{ij}$  - unit waste transportation cost from  $i$  to  $j$

$Q_j$  - capacity of a landfill at potential location  $j$

$R_1$  - minimal acceptable separation distance between any two located landfills

$R_2$  - separation distance between any landfill location and end users

$N_j = \{k \in J \mid d_{jk} < R_1, j \neq k\}, \forall j \in J$  - set of potential locations that are on distance less than  $R_1$  from particular location  $j$ , excluding itself

$\Pi_i = \{j \in J \mid d_{ij} < R_2\}, \forall i \in I$  - set of potential locations that cover end user  $i$  within  $R_2$

$M$  - large positive number

Variables:

$$x_j = \begin{cases} 1, & \text{if landfill is located at potential location } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ij} = \begin{cases} 1, & \text{if end user } i \text{ is associated to landfill } j \\ 0, & \text{otherwise} \end{cases}$$

$z_i$  - number of landfills that cover end user node  $i$

Formulation of the problem:

$$\min OF_1 = \sum_j f_j x_j + \sum_i \sum_j w_i c_{ij} d_{ij} y_{ij} \quad (1)$$

$$\min OF_2 = \sum_i v_i z_i \quad (2)$$

s.t.

$$Mx_j + \sum_{k \in N_j} x_k \leq M, \forall j \in J \quad (3)$$

$$\sum_j y_{ij} = 1, \forall i \in I \quad (4)$$

$$\sum_i w_i y_{ij} \leq Q_j x_j, \forall j \in J \quad (5)$$

$$\sum_{j \in \Pi_i} x_j = z_i, \forall i \in I \quad (6)$$

$$x_j \in \{0,1\}, y_{ij} \in \{0,1\}, z_i \in N_0 \forall i \in I, j \in J \quad (7)$$

In the objective function (1), the aim is to minimize total facility and transportation costs, i.e. the total cost of facility establishing and entire demand satisfaction. Second objective function (2), minimizes the total number of end users undesirably influenced by landfills. This objective function is created to account for multiple landfill locations covering an end user, e.g. if some end user's node is covered by two landfills then its population will be counted twice in the objective function because the impact on that population is doubled. Constraints (3) are characteristic for Anti-covering location problem (ACL<sub>P</sub>) (Moon and Chaudhry, 1984) and MCLPDC. They are referred as Neighborhood Adjacency Constraints. If node  $j$  is selected for facility placement (i.e.  $x = 1$ ), then the term  $Mx_j$  equals the right hand side term  $M$  and forces  $\sum_{k \in N_j} x_k = 0$ . Thus, if site  $j$  is

used, then all sites  $k$  within the  $R_1$  distance neighborhood of site  $j$ ,  $N_j$ , are restricted from use. Those constraints are practically oriented aiming at landfills dispersion. Constraints (4) and (5) are regular FCLP constraints, where (4) guarantee that each end user is served from one facility, while constraints (5) play a double role: ensure that the capacity of facilities is not exceeded and prevent users from being allocated to non-open facilities. With constraints (6), number of facility locations that negatively affect particular end user, i.e. the gap between them is less than predefined separation distance  $R_2$ , are determined. Consequently those constraints determine which users are covered and by how many landfills. Constraints (7) describe problem variables. Other specific constraints could be added according to the specific problem.

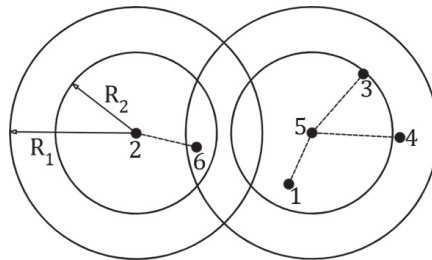


Figure 1. Illustration of separation distances for the numerical example

### 3. NUMERICAL EXAMPLE

In this section, we tested proposed bi-objective model for landfill locating on one illustrative example. The observed area consists of 6 nodes which are simultaneously potential landfill sites and end users. All inputs are illustrative and time based costs are normalized at daily level. Daily quantity of waste generated per person at end user  $i$  ( $q_i$ ) is adopted to be 0.8 kg for all  $i \in I$ , while the capacity of landfill sites is put to be sufficient enough that cannot be exceeded in this example. Let transportation unit waste cost be 0.8 €/km per truck load between all  $i \in I$  and  $j \in J$ . If we assume the truck capacity of 10000 kg then  $c_{ij} = 0.00008$  €/km·kg. Fixed cost ( $f_j$ ) of locating a landfill is uniform for all  $j \in J$  and reduced to a daily amount of 1500 €. In Table 1, input parameters for proposed bi-objective model for landfill location are presented. Value of big  $M$  is set to 10. Values of  $R_1$  and  $R_2$  are set to 250 and 160 km, respectively.

Problem was developed using Python 2.7 programming language and solved by Cplex 12.6 software (IBM, 2012). Results for numerical example including all Pareto optimal solutions are presented on Figure 2. Marginal solution for  $OF_1$  is  $x = (0, 1, 0, 0, 1, 0)$  and  $OF_1 = 9,680.46$  €, while marginal solution for  $OF_2$  is  $x = (0, 0, 0, 1, 0, 0)$  and  $OF_2 = 208,895$  residents. For the marginal solution for  $OF_1$  (Figure 1),  $OF_2$  is 1,192,758. Next Pareto solution is  $(1, 1, 0, 0, 0, 0)$  and  $OF_1 = 11,200.52$  while  $OF_2 = 1,074,463$ , and so on until marginal solution for  $OF_2$  where  $OF_1$  is 22,682.42. All those solutions are valuable for the decision maker because they provide complete insight into the necessary information in finding the most preferred solution when deciding on landfills'

locations. In this small scale example it was possible to find all Pareto solutions, which is difficult for larger and almost impossible for real world data cases. It is usual that the decision maker has preferences for certain objectives (e.g. costs), so one solving approach could be relaxed lexicographic method to support such preference expressed by their order. If the decision maker chose in this model  $OF_1$  as preferable objective then if for example he/she allows the total cost to increase for 23.5% from the optimal  $OF_1$  (from 9680.46 to 11,955.01 €),  $OF_2$  could decrease for 40.7% (from 1,192,758 to 707,529 residents).

Table 1. Values of the input parameters:  $d_{ij}$ ,  $v_i$  and  $w_i$

$i/j$	1	2	3	4	5	6	$v_i$	$w_i$
1	0	366.89	268.85	283.33	111.38	178.19	284929	227943
2	366.89	0	421.23	435.8	317.13	123	187581	150065
3	268.85	421.23	0	132.99	158.14	313.76	118295	94636
4	283.33	435.8	132.99	0	173.07	329.27	90600	72480
5	111.38	317.13	158.14	173.07	0	210.71	290900	232720
6	178.19	123	313.76	329.27	210.71	0	311053	248842

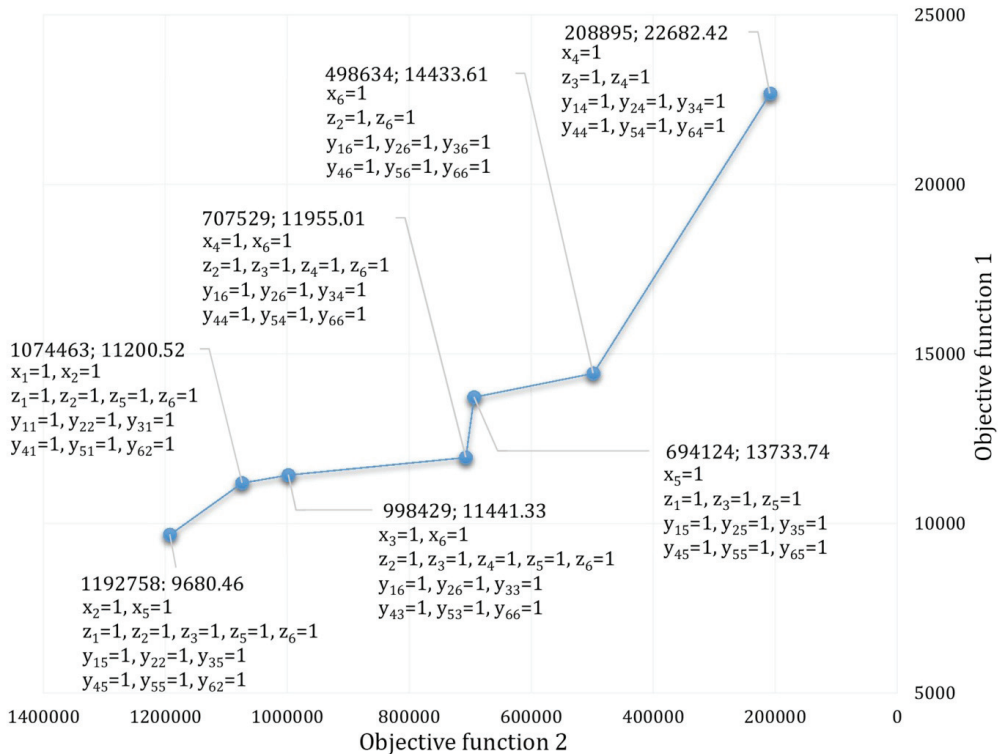


Figure 2. Pareto optimal solutions for numerical example

#### 4. CONCLUDING REMARKS

This paper presents bi-objective model for determining landfill locations, in which the most common cost minimization function is utilized as first objective and as second objective function we minimized number of end users influenced by the negative impact of the landfills. This model

combines approaches and ideas from several location problems, such as FCLP, ACLP and MCLPDC. We presented one numerical example, with all Pareto optimal solutions and an illustration of the relaxed lexicographic method usage. The model should be understood as the beginning of our research in this area. Future research will focus on real world data problem testing, involving different approaches for problem solving, as well as on model upgrading to include location of transfer stations etc. We believe that proposed model has potentials and can be beneficial for the government organizations, local authorities as well as other organizations related to waste management issues.

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