

## OPTIMIZATION OF FLEET SIZE WITH BALANCED USE OF VEHICLES: CASE OF SUGAR BEET TRANSPORTATION

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**Abstract:** *In order to make sugar beet collection and transportation processes efficient and economical, it is needed to define optimal fleet size and to define tours which are suitable to be realized by the same vehicle. Mentioned task of assigning particular tours to a certain vehicle corresponds to the problem of "packing tours to the vehicles", that can be easily recognized as a well known one-dimensional bin-packing problem. In case of sugar beet transportation we are not only interested in determining schedules providing sugar beet supply with the fewest number of vehicles, but also in providing well-balanced vehicle schedules. Therefore, in this paper we analyze possibilities for fleet size optimization considering equity and fairness criteria. We propose modeling approach and some numerical results obtained from the model application.*

**Keywords:** *Vehicles scheduling, Fleet sizing, Bin packing, Load balancing.*

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### 1. INTRODUCTION

Sugar beet is well known industrial culture which is widely used for sugar production in Europe, and particularly in Serbia. The sugar beet harvesting period, which usually lasts two or three months, depending on the weather conditions, starts from September or October. In this period, known as campaign, transport demand is very high, and in average, in the case of Serbia, cca 100 vehicles need to make more than 200 tours between storage piles of harvested beet and a sugar mill every day. This huge transport demand is usually served by different 3PL providers, hired by the sugar company. In the process of collection and transportation of sugar beet different companies are engaged, some with small, and some other with large vehicle fleets. Depending on the distance between a sugar mill and sugar beet storage piles, but also depending on the length of the queues in front of the mill, vehicles make between one and several tours (usually up to five) during the working day lasting 24 hours.

All vehicles, i.e. transport companies, are hired on the basis of the predefined schedules including locations of sugar beet storage piles, quantities and required number of tours for any particular day in the planning period of three to six days, while those short term plans are based on the monthly or plans for the whole period of campaign. Hence, to make sugar beet collection and transportation processes efficient and economical, and to schedule and hire vehicles optimally, it is needed to define fleet size to be hired, and consequently, to define tours which are suitable to be realized by the same vehicle. Mentioned task of assigning particular tours to a certain vehicle, which obviously corresponds to the problem of "packing tours to the vehicles", can be easily recognized as a well known one-dimensional bin-packing problem (BPP). The BPP

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consists of packing objects of different sizes into a finite number of similar bins (containers), in a way that the number of used bins is minimized (Trivella and Pisinger 2016). This problem is one of the most famous in combinatorial optimization which has been studied since the 1939, when Kantorovich, among the other problems in organizing and planning production, considered scrap minimization, which corresponds to the BPP. His study, originally written in Russian, later is translated and published in English (Kantorovich 1960). Since then, the problem and its extensions continually occupy research interest, so the literature on these problems is huge. This problem was also paradigm for new approaches to the analysis of approximation algorithms and, because the problem is strongly NP-hard, many heuristic and metaheuristic approaches have been proposed. To the more interested reader we propose excellent research report of Delorme et al. (2015).

The bin-packing problem has very wide application in different areas and, as it is mentioned above, one possible application is in the fleet size optimization. This application is intuitively clear and corresponds to the problem of finding minimal number of vehicles needed for realization of all tours visiting defined set of nodes, when each vehicle performs sequence of assigned tours during its working time. However, as it is correctly stated by Trivella and Pisinger (2016), sometimes "we are not only interested in determining a packing with fewest bins, but also obtaining well-balanced packings". It is of particular importance in defining vehicles' schedules for the case of sugar beet collection and transportation, because vehicle tours of all hired vehicles should be based on equity and fairness principles. This means that the predefined schedules should be balanced, including as similar as possible tours' sequences, with similar transportation distances and collected quantities.

In general, literature offers different equity objectives and work load balancing criteria which are applicable for different types of vehicle routing, fleet size determination and bin packing problems as it can be seen in Matl et al (2016), Trivella and Pisinger (2016), Cossari et al. (2012), etc. Some of those approaches are based on one criterion, while the others are multidimensional. Also, to measure workload, different measures are proposed. Ho et al. (2009), and recently, Cossari et al. (2012), used criterion based on square errors, that they called "the normalized sum of square for workload deviations (NSSWD)", which is quite logical to be used from the statistical perspective, but because it is non linear, needs appropriate heuristics. Since the equity and fairness principles for the case of sugar beet transportation include two parameters: transportation distances and collected quantities, our approach in its nature is bi-criteria. In our model we include those parameters as constraints, keeping the model linearity by restricting absolute difference of equity measures (traveled distance and collected quantity), between every pair of vehicles, within a defined threshold. We consider two possible approaches. One based on partial consideration of the traveled distance and collected quantity, and another based on compound measure which corresponds to freight carriage unit *tkm*.

Remaining part of the paper is organized as follows. The problem description is given in Section 2. Proposed modeling approach and mathematical formulation is described in Section 3. Section 4 presents test instances and computational results. Concluding remarks are given in Section 5.

## 2. PROBLEM DESCRIPTION

Supply area of a sugar mill includes numerous sugar beet fields, and lot of potential locations for the storage piles. During the campaign, harvested beet is brought to those locations which then become sugar beet supplying nodes. Quantities of sugar beet stored on those locations can vary considerably, but it is always required to be multiple visited by collection vehicles. Supply nodes, i.e sugar beet storage piles are spread in the mill supply area on different distances, usually in the range of 5 to 50 kilometres, so that collection vehicles visiting storage locations close to the mill are able to make several tours, four to five, while those visiting farthest location make only one of two tours during the day. During the campaign, sugar mill has more or less constant

production rate, which needs constant supply of sugar beet which means realization of required vehicle tours every day. Daily vehicles' schedule aims to provide required supply intensity, while respecting different criteria like weather conditions, sugar beet freshness, vehicle capacity utilization, fairness and equity principles, etc. Freshness of the sugar beet stored on piles at different locations is very important because it longer stay means deterioration and lesser sucrose content which, together with weather and road conditions in a supply area, can be considered as primary criterion defining storage piles to be visited during a day.

Defining the vehicles' schedule which minimizes a needed fleet size to be hired, and provide required beet collection intensity from the set of predefined storage piles locations, while respecting available working time of vehicles as well as equity and fairness in tours assignment, is the second planning phase, and the problem considered in this study. Transport network which represent the problem considered in this study is shown in the Figure 1.

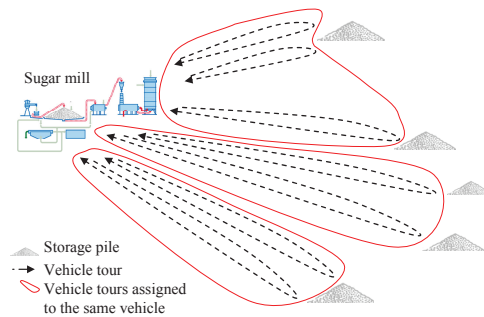


Figure 1. Sugar beet supply network

If there are  $M$  tours, and each lasts  $t_i$  time units, where  $i \in M$ , and each vehicle  $j \in N$  is available during its working period  $T_j$ , then the problem of finding minimal number of vehicles  $N^*$  needed for realization of all tours, when each vehicle performs sequence of assigned tours during its working time, corresponds to the solution of the bin-packing problem. Note that, without loss of generality, it is possible to remunerate all tours visiting different storage piles locations, making  $M$  tours where some visit the same location. Fairness and equity in tours assignment are respected here through the balancing of the total traveled distance and quantity collected by vehicles. More precisely, we balance total time needed for realization of all tours assigned to a vehicles, as well as number of tours vehicle performs, because the quantity collected in a tour can be assumed to be equal.

### 3. MODELING APPROACH AND MATHEMATICAL FORMULATION

To formulate the problem we model the BPP in the usual form as an Integer Linear Program (ILP) with two binary decision variables  $x_j$ , and  $y_{ij}$ . Binary variable  $x_j$  equals 1 if vehicle  $j \in N$  is used in the solution and it is equal to 0 otherwise. Binary variable  $y_{ij}$  equals 1 if vehicle  $j \in N$  performs the tour  $i \in M$ , where  $M$  and  $N$  respectively represent sets of collection tours and vehicles. The proposed model is formulated as follows.

$$\text{Min} \sum_j x_j \quad (1)$$

$$\sum_{i \in M} y_{ij} \cdot t_i \leq x_j T_j \quad \forall j \in N \quad (2)$$

$$\sum_{j \in N} y_{ij} = 1 \quad \forall i \in M \quad (3)$$

$$y_{ij} \in \{0,1\} \quad x_j \in \{0,1\} \quad \forall i \in M, \forall j \in N \quad (4)$$

The objective function (1), tries to minimize vehicle fleet size, while the constraints (2) impose that the working time  $T_j$  of any used vehicle  $j \in N$  is not exceeded by assigned set of tours each lasting  $t_i$  time units. Constraints set (3) ensure that all tours are realized, but only once. Constraints (4) define variables domains.

To introduce fairness and equity principles in tours assignment, we extend standard form of the BPP model (1) - (4) with additional vehicles' workload balancing constraints. We analyze two concepts of vehicles' workload balancing constraints. In the first concept we use two sets of constraints (3a) and (3b). Constraints (3a) restrict absolute difference of the total time needed for realization of all tours assigned to the vehicles, between every pair of vehicles  $u, v \in N$  used in the solution. This difference should not be greater than the given "time difference threshold" -  $\tau$ . Similarly, constraints (3b) restrict absolute difference of the total number of tours assigned to the vehicles, between every pair of vehicles  $u, v \in N$  used in the solution, This difference should not be greater than the given "number of tours difference threshold" -  $\delta$ .

$$\left| \sum_{i \in M} y_{iu} \cdot t_i - \sum_{i \in M} y_{iv} \cdot t_i \right| \leq \tau + M_1(1 - x_u) + M_2(1 - x_v) \quad \forall u, v \in N, u \neq v \quad (3a)$$

$$\left| \sum_{i \in M} y_{iu} - \sum_{i \in M} y_{iv} \right| \leq \delta + M_3(1 - x_u) + M_4(1 - x_v) \quad \forall u, v \in N, u \neq v \quad (3b)$$

In the second concept of defining vehicles' workload balancing constraints, our idea was to use compound measure which corresponds to freight carriage unit  $tkm$ . However, because the quantity collected in a tour is assumed to be equal for all vehicles, it is enough to consider only total travelled distance. Since the travel distance corresponds to the travel time, as an estimate of difference in  $tkm$  realized by different vehicles, we used above given set of constraints (3a).

Based on previous consideration, to model the fairness and equity principles in tours assignment, accordingly to the first concept we used the model (1) - (4), with additional constraints (3a) and (3b), while to model the second concept we used the model (1) - (4), extended only with constraints (3a). Values  $M_1, M_2, M_3$  and  $M_4$  are large enough constants (big  $M$ ), used to introduce "and" statement in the constraints (3a) and (3b), since the threshold values  $\tau$ , and  $\delta$  are only applied when tours, performed by both vehicles  $x_u$ , and  $x_v$ ,  $\forall u, v \in N, u \neq v$ , are in the solution. Note that shown form of the constraints (3a) and (3b) include absolute values of the differences which make the model nonlinear. However, those expressions can be easily linearized by standard transformation of the absolute values, which in total makes four sets of inequalities.

#### 4. COMPUTATIONAL RESULTS

To verify proposed approach, we tested the model on several problem instances. Because the BPP is strongly NP-hard, our problem instances are smaller than the real world problems' sizes. Supply area has 10 storage pile locations which are randomly distributed around the sugar mill, on the distances which correspond to the tours' realization time randomly generated from the uniform distribution  $U(2,8)$ . Total daily transportation demand was 1000 t of a sugar beet. Values of constants  $M_1$ , and  $M_2$  are assumed to be equal to the vehicles availability period of 24 time units, which is the same for all vehicles. Values of constants  $M_3$ , and  $M_4$  are assumed to be equal to the total number of the vehicles' tours  $|M|$ . Results of the model application are shown in the Table 1. The model was implemented in CPLEX 12.2, on 64 bit HP desktop, 3.20 GHz Intel Core i5-3470 with 8 GB RAM memory.

Table 1. Impact of the balanced and non balanced tours assignment

Problem instance	Number of vehicle tours (T) and average vehicles' usage (U) <b>Without balance</b>			
	Min T	Max T	Min U	Max U
	1	3	5	0.851
2	3	5	0.842	0.990
3	3	4	0.471	0.985
4	3	8	0.521	0.990
5	3	5	0.805	0.996
6	2	5	0.346	0.986
7	4	5	0.806	0.971
8	5	7	0.881	0.964
9	3	5	0.863	0.998
10	3	6	0.287	0.986
<b>Average</b>	<b>3.2</b>	<b>5.5</b>	<b>0.667</b>	<b>0.983</b>
<b>Min/max</b>	<b>2</b>	<b>8</b>	<b>0.287</b>	<b>0.998</b>

Problem instance	Number of vehicle tours (T), and average vehicles' usage (U) - <b>Balanced (3a+3b)</b>															
	$\tau=\min\{t_i\}, \delta=1$				$\tau=\min\{t_i\}, \delta=2$				$\tau=2, \delta=1$				$\tau=2, \delta=2$			
	Min T	Max T	Min U	Max U	Min T	Max T	Min U	Max U	Min T	Max T	Min U	Max U	Min T	Max T	Min U	Max U
1	4	4	0.829	0.960	3	5	0.850	0.988	4	4	0.855	0.932	3	5	0.850	0.932
2	4	4	0.828	0.960	4	4	0.805	0.984	4	4	0.857	0.938	3	5	0.852	0.933
3	3	4	0.808	0.965	3	4	0.808	0.965	3	4	0.843	0.923	3	4	0.871	0.945
4	4	5	0.853	0.942	4	6	0.571	0.926	4	5	0.852	0.926	4	6	0.845	0.921
5	3	4	0.786	0.979	3	4	0.853	0.972	3	4	0.890	0.972	3	4	0.883	0.965
6	3	4	0.830	0.986	3	4	0.830	0.985	3	4	0.830	0.876	3	5	0.859	0.939
7	4	5	0.845	0.995	4	5	0.882	0.970	4	4	0.804	0.885	4	4	0.805	0.883
8	5	6	0.885	0.966	5	6	0.885	0.974	5	6	0.887	0.964	5	6	0.894	0.974
9	4	4	0.863	0.992	3	5	0.841	0.992	4	4	0.890	0.949	3	5	0.885	0.963
10	4	5	0.814	0.918	3	5	0.838	0.945	4	5	0.838	0.919	4	5	0.838	0.917
<b>Average</b>	<b>3.8</b>	<b>4.5</b>	<b>0.834</b>	<b>0.966</b>	<b>3.5</b>	<b>4.8</b>	<b>0.817</b>	<b>0.971</b>	<b>3.8</b>	<b>4.4</b>	<b>0.855</b>	<b>0.929</b>	<b>3.5</b>	<b>4.9</b>	<b>0.858</b>	<b>0.938</b>
<b>Min/max</b>	<b>3</b>	<b>6</b>	<b>0.808</b>	<b>0.995</b>	<b>3</b>	<b>6</b>	<b>0.571</b>	<b>0.992</b>	<b>3</b>	<b>6</b>	<b>0.804</b>	<b>0.972</b>	<b>3</b>	<b>6</b>	<b>0.805</b>	<b>0.974</b>

Instances	Number of vehicle tours (T), and average vehicles' usage (U) - <b>Balanced (3a)</b>							
	$\tau=\min\{t_i\}$				$\tau=2$			
	Min T	Max T	Min U	Max U	Min T	Max T	Min U	Max U
1	3	5	0.831	0.960	3	5	0.850	0.932
2	4	4	0.813	0.990	3	5	0.853	0.934
3	3	4	0.808	0.965	3	4	0.852	0.922
4	3	7	0.842	0.937	3	8	0.845	0.927
5	3	4	0.808	0.973	3	4	0.876	0.958
6	3	4	0.830	0.986	3	5	0.859	0.940
7	4	5	0.882	0.994	4	4	0.805	0.883
8	5	7	0.869	0.990	5	7	0.886	0.964
9	3	5	0.842	0.992	3	6	0.885	0.964
10	4	6	0.818	0.937	4	5	0.838	0.917
<b>Average</b>	<b>3.5</b>	<b>5.1</b>	<b>0.834</b>	<b>0.972</b>	<b>3.4</b>	<b>5.3</b>	<b>0.855</b>	<b>0.934</b>
<b>Min/max</b>	<b>3</b>	<b>7</b>	<b>0.808</b>	<b>0.994</b>	<b>3</b>	<b>8</b>	<b>0.805</b>	<b>0.964</b>

Obviously, reported results show that the proposed concept of introducing fairness and equity in tours assignment, based on defined balancing criteria provide very good results, where the compound criterion gives slightly wider range of the average assigned number of tours, and average vehicle capacity usage, while maximum and minimum values are very close. This also means that the proposed approach and defined criteria should be considered as good candidates

that could provide efficient balancing mechanism to control fairness in defining vehicles' schedules.

## 5. CONCLUSION

In this paper we analyze the idea of introducing equity and fairness principles for the case of sugar beet transportation. The proposed approach to defining balanced schedule of vehicles used in sugar beet supply is based on two parameters: transportation distances and collected quantities. In the model we include these parameters as constraints keeping the model linearity by restricting absolute difference of equity measures between every pair of vehicles, within a defined threshold. Two possible approaches have been considered. One based on two measures: traveled distance and collected quantity, and another based on single compound measure which corresponds to freight carriage unit *tkm*.

Results of numerical experiments are very promising, since the model give expected results, keeping the workload balance, while simultaneously determining the minimal fleet size. However, because BPP is strongly NP-hard, numerical experiments were limited to smaller instances. The fact that the real word problems of this type are much larger, immediately opens one of the most important future extension of this research which is related to development of appropriate heuristics and metaheuristic approaches able to respond to sugar beet supply system of real size. Other possible research directions are in the field of the further analysis of defined balancing criteria and their comparison with other possible fairness and equity measures.

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