SPATIAL COMPETITION WITH REGULATORY INTERVENTION

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Abstract: *A Competition model involves situations where two or more entities (companies) compete with the purpose of gaining market share. In a region with a certain spatial structure we consider spatial competition models, and such location-allocation models fall under the game theory. The paper focuses on modeling such a situation when two competing companies offer identical goods on the market, but the price of these goods may vary. These companies are deciding where to build their operations. The cost of the customer includes both the price of the goods and the transport costs. However, the regulator also enters into the game with a preference of a specific location in order to support a local economy, reduce negative environmental impacts, etc. As a result, customers can be divided into different groups. One group consists of myopic customers who only consider their own costs, whilst the second group includes customers who follow regulator recommendations and buy only in the preferred location. The third group consists of customers who change their behavior based on their maximum cost.*

Keywords*: Spatial competition, regulator preference, game theory*

1. INTRODUCTION

The paper is focused on location-allocation models that are parts of spatial competition models. We will model the location of two competing companies offering an identical product at different prices in a geographically determined market. The first known models are associated with the name of Hotelling (Hotelling, 1929). However, Hotelling considered the location along the line and we will extend these considerations by adding a spatial structure that can be described as a graph. We will further extend the model by situation where a regulator wants to actively intervene and prefer the operations at a particular location. Obviously, the regulator's preference does not always depend solely on economic benefits, but may be a purely political decision to support certain areas.

The easiest way to take into account node preference would be to regulate the price (some form of subsidy or penalty). However, we will consider also the company's awareness and suppose that some customers will follow the regulator's recommendation and buy at the preferred node, thereby increasing demand at that node. Thus, a society (consumers) can

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be divided into two groups. In the first case, consumers are considered to be myopic (short-sighted), that is, they are unaware of the damage that occurs when purchasing goods (and building an establishment) at a location other than the designated location. Such behaviour can cause negative externalities because of, for example, higher traffic loads, which causes higher pollution of the entire area and not just at a given location. If the regulator's preference for a given node is to support local manufacturers or suppliers, ignoring preferred nodes can have a negative impact on the economic situation of the entire region.

One of the major factors in preferred areas is the application of conscious zoning (Sequeira Lopez, 2018). Zoning is used as a way to isolate negative externalities (such as pollution) outside public areas or in the case of product (service) support in the preferred area (Pekár J. et all, 2012). On the other hand, customers who follow the recommendations of the regulator support the local economy and can also generate a lower negative impact on the environment.

The regulator assumes that the market will be divided among conscious and unconscious customers. In this case, conscious customers will buy in the preferred node. Demand of unconscious customers will be shared between the preferred and non-preferred node.

We will solve this situation within the game theory models and in the simplest case it is possible to formulate a game with a constant sum. Therefore, we will introduce the basic formalization of such a game.

2. TWO-PLAYER GAME WITH CONSTANT SUM

A two-player game with constant sum can be described as follows:

Let $P = \{1,2\}$ be a set of players. Each player has a finite set of strategies (X – player 1, Y – player 2}, i.e. player 1 chooses $\mathbf{x} \in X$, player 2 chooses $\mathbf{y} \in Y$, then $(x, y) \in X\text{x}Y$ is set of all results of the game $(x, y) \in X\text{x}Y$. The individual elements of the sets X and Y can be arranged by a finite number of non-negative numbers (elements of the set $X: i = 1,2, \dots m$ and elements of the set $Y: j = 1,2, \dots n$) and the results of the game for player 1 can be indicated in the matrix $\mathbf{A}_{mxn} = \{a_{ij}\}$, where a_{ij} indicates the player's payoff at the result (i, j). The values of the game for player 2 can be indicated in the matrix $\mathbf{B}_{nxm} = \{b_{ji}\}$, where b_{ji} indicates the player's payoff at the result (j, i). The constant sum can be characterized as follows: $\mathbf{B}_{nxm} = \mathbf{C}_{nxm} - \mathbf{A}^{\mathrm{T}}_{mxn}$, where $\mathbf{C}_{nxm} = \{c\}$ with c being the constant independent of the strategy choice.

The aim is to identify equilibrium strategies for both players. The status is defined as equilibrium, if the system has a tendency to remain in such state under certain conditions (only such set of strategies can be considered as a satisfactory result, if any effort to unilaterally violation automatically leads to damage to a player attempting to do so).

Game solutions are based on the following assumptions: Both players have complete information about the model of the conflict situation, i.e. they know the payoff matrix $\mathbf{A}_{mxn} = \{a_{ij}\}$, players are intelligent, i.e. the players want to maximize the payout and know that so does the opponent, the players are careful, i.e. they try to minimize the risk. The solution to the game results in identifying the equilibrium point in pure strategies (saddle point of matrix $\mathbf{A}$) or in mixed strategies (Chobot et al, 1991). (Goga, 2013), (Dlouhý, 2007).

The mixed strategy of player 1 is the m-dimensional vector $x, \sum_{i=1}^{m} x_i = 1, x_i \geq 0, i =\leftarrow V$ and the mixed strategy of player 2 is the n-dimensional vector$y, \sum_{j=1}^{n} y_j = 1, y_j \geq 0, j \leftarrow V$. Mixed strategies can then be identified using simple linear programming problem (Chobot et al, 1991).

3. SPATIAL COMPETITION MODEL WITH REGULATORY INTERVENTION

The spatial game is based on the following assumptions: Let $V = \{1,2, \dots n\}$ is a set of customers and let be given a finite continuous oriented edgewise-rated graph $G = (V, H)$, where V represents a non-empty n-element set of graph nodes, and $H \subset VxV$ represents a set of edges $h_{ij}, i, j \in V$ from the i-th to j-th., with each oriented edge h_{ij} being assigned a real number $o(h_{ij})$ called a valuation or also the edge value of h_{ij}. The network game is formulated in a so-called complete or a complete weighted graph $\bar{G} = (V, \bar{H})$ with the same set of nodes as graph G, where $\bar{H}$ is the set of edges between each pair of nodes i and j, their valuation being equal to the minimum distance between the nodes i and j in the original graph $i, j \in V$. If d_{ij} represents the minimum distance (the shortest path length) between nodes i and j, then the matrix $\mathbf{D}_{nxn} = \{d_{ij}\}$ is the shortest distance matrix.

Let´s assume two companies (players), $P = \{1,2\}$ offering a homogeneous product (goods or service) that have the ability to build their operations in one of the nodes of graph $\bar{G}$. Suppose the nodes of graph also represent the seat of the customers with constant demand. Although both players offer an identical product in an unlimited amount, the product price is different. Let us denote $p^{(1)}$ the product price for player 1 and $p^{(2)}$ the product price for player 2. We do not consider any capacity limitations; every customer can buy a product at any company. Customers, however, take into consideration the total price of the product consisting of both the purchase price of the product and the price of the transport to a chosen company. Transport costs are rated as $t/$unit of distance. The aim is to identify those nodes in which companies build their operations, assuming mutual interaction, and it is known that customers always prefer lower cost purchases (in case of equal costs, companies will split demand in half). The model taking into account the above assumptions was presented in (Sequeira Lopez & Čičková, 2018). In this way, the cost of the consumer of purchasing at company 1 can be written in a matrix $\mathbf{N}^{(1)} = \{n_{ij}^{(1)}\}, i, j \in V$ with elements are defined as follows:

$$n_{ij}^{(1)} = t * d_{ij} + p^{(1)}, i, j \in V \qquad (1)$$

Analogical for the player 2 we specify the matrix $\mathbf{N}^{(2)} = \{n_{ij}^{(2)}\}, i, j \in V$:

$$n_{ij}^{(2)} = t * d_{ij} + p^{(2)}, i, j \in V \qquad (2)$$

Now, consider the influence of the regulator, which aims to actively increase interest in a particular node. The models are further expanded to include situations where the regulator wants to actively disable or reduce consumption at a specific node. The regulator may interfere by affecting the price at a given node through a fine or penalty at the node where the location of the establishment is undesirable from its perspective. Thus, the total cost to the consumer at a given node is increased, provided that the company takes these measures into account in the price of the product, that is, they increase the product price by the amount of the regulator's sanction.

Let´s introduce a parameter $\lambda \in (0,1)$ that represents awareness of the society. It means that λ % of the society will be aware and will follow the preferences of the regulator (will purchase in preferred location) regardless its own costs. Let´s denote a preferred location as $pref \in \{1,2, \dots n\}$. Then, the myopic consumers are represented by $1-\lambda$.

Then it is possible to define the elements of the payoff matrix of player 1 ($\mathbf{A}$) in the form of the following pseudo code:

SET PAREMETERS $V = \{1,2, \dots n\}, \boldsymbol{D}_{nxn} = \{d_{ij}\}, i,j \in V, t, p^{(1)}, p^{(2)}, \lambda \in (0,1),$

$pref \in V$

DECLARE $N_{n*n}^{(1)} = \{n_{ij}^{(1)}\}, i,j \in V;\ A_{n*n}\{a_{ij}\}, i,j \in V;$

LOOP $(i,j \in V)$ **DO**

$n_{ij}^{(1)} = t * d_{ij} + p^{(1)};$

$n_{ij}^{(2)} = t * d_{ij} + p^{(2)};$

$a_{ij} = 0;$

LOOP $(k,i,j \in V)$ **DO**

IF $n_{ki}^{(1)} < n_{kj}^{(2)}$ **and** $i = pref$ **DO**$a_{ij} = a_{ij} + 1;$

ELSEIF $n_{ki}^{(1)} < n_{kj}^{(2)}$**and** $i \neq pref$ **DO**$a_{ij} = a_{ij} + (1-\lambda);$ (3)

ELSEIF $n_{ki}^{(1)} > n_{kj}^{(2)}$ **and** $j = pref$ **DO**$a_{ij} = a_{ij} + \lambda;$

ELSEIF $n_{ki}^{(1)} = n_{kj}^{(2)}$ **and** $j = pref$ **DO**$a_{ij} = a_{ij} + \lambda + 0{,}5(1-\lambda);$

ELSEIF $n_{ki}^{(1)} = n_{kj}^{(2)}$ **DO**$a_{ij} = a_{ij} + 0.5;$

ENDIF

Obviously, under the given assumptions, it is possible to formulate a game with a constant sum, where the constant of the game $c = n$ (since players share a constant demand of n nodes) and elements of matrix $\mathbf{B}$ can be calculated as $\mathbf{B}_{nxn} = \mathbf{C}_{nxn} - \mathbf{A}_{nxn}^{\mathrm{T}}$, where $\mathbf{C}_{nxn} = \{c\}$.

Now, consider the maximum difference cost that the consumer is willing to spend on the goods. Let´s denote such maximum difference cost as T_{MAX}. Then, the demand is shared between the preferred and non-preferred node based on maximum difference costs, and the conscious consumer will also shop at the non-preferred node if the total preferred node purchase cost exceeds T_{MAX}.

Then it is possible to define the elements of the payoff matrix of player 1 ($\mathbf{A}$) in the form of the following pseudo code:

SET PAREMETERS $V = \{1,2, \dots n\}, \boldsymbol{D}_{nxn} = \{d_{ij}\}, t, p^{(1)}, p^{(2)}, \lambda \in (0,1), pref \in V, T_{MAX}$

$N_{n*n}^{(1)} = \{n_{ij}^{(1)}\}, i,j \in V;\ A_{n*n}\{a_{ij}\}, i,j \in V;$

LOOP $(i,j \in V)$ **DO**

$n_{ij}^{(1)} = t * d_{ij} + p^{(1)};$

$n_{ij}^{(2)} = t * d_{ij} + p^{(2)};$

$a_{ij} = 0;$

LOOP $(k, i, j \in V)$ **DO**

IF $n_{ki}^{(1)} < n_{kj}^{(2)}$**and** $= pref$ **DO** $a_{ij} = a_{ij} + 1;$

ELSEIF $n_{ki}^{(1)} < n_{kj}^{(2)}$**and** $i \neq pref$

and $\left(n_{kj}^{(2)} - n_{ki}^{(1)}\right) \leq T_{MAX}$**DO** $a_{ij} = a_{ij} + (1 - \lambda);$ (4)

ELSEIF $n_{ki}^{(1)} > n_{kj}^{(2)}$ **and** $i = pref$

and $\left(n_{ki}^{(1)} - n_{kj}^{(2)}\right) \leq T_{MAX}$ **DO** $a_{ij} = a_{ij} + \lambda;$

ELSEIF $n_{ki}^{(1)} = n_{kj}^{(2)}$ **and** $j = pref$ **DO**$a_{ij} = a_{ij} + \lambda + 0{,}5(1 - \lambda);$

ELSEIF $n_{ki}^{(1)} = n_{kj}^{(2)}$ **DO** $a_{ij} = a_{ij} + 0.5;$

ENDIF

In the next section, we will illustrate this approach by solving illustrative examples.

4. EXAMPLES

Let's assume the existence of five potential customers $V = \{1,2,3,4,5\}$, each of them is located in the unique node of a graph G. We also assume the form of a duopoly market where each of the companies can build a branch office in any node of this graph $i \in V$. Each player (company) aims to maximize the number of nodes that are served. Although both players offer a homogeneous product, its price is $p^{(1)}$= 1 for player 1 and $p^{(2)}$= 1.1 for player 2. Let consider the following weighted graph in figure 1:

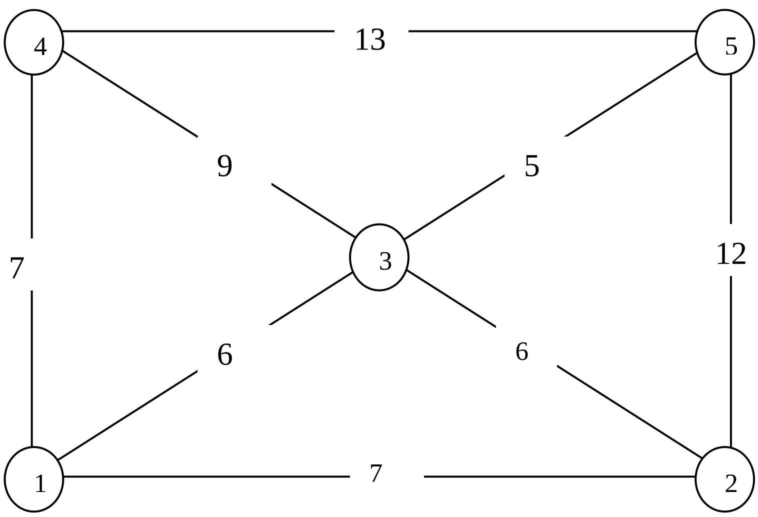

Figure1. Graph of distances between nodes

Let the shortest distance matrix $\mathbf{D} = \{d_{ij}\}, i, j \in V$ between all the nodes of the network are represented by the shortest distance matrix for ours five nodes.

$$\mathbf{D} = \begin{bmatrix} 0 & 7 & 6 & 7 & 11 \\ 7 & 0 & 6 & 14 & 11 \\ 6 & 6 & 0 & 9 & 5 \\ 7 & 14 & 9 & 0 & 13 \\ 11 & 11 & 5 & 13 & 0 \end{bmatrix}$$

We assume unit transport costs $t = 1$, Let´s suppose the regulator prefers the node 3, let's say the parameter $\lambda \in (0,1)$ is 0.4, i.e. 40% of the population will follow the regulator strategy. Based on (1) and (2) it is possible to quantify matrix elements $\mathbf{N}^{(1)}$ and $\mathbf{N}^{(2)}$ as follows:

$$\mathbf{N}^{(1)} \begin{bmatrix} 1 & 8 & 7 & 8 & 12 \\ 8 & 1 & 7 & 15 & 12 \\ 7 & 7 & 1 & 10 & 6 \\ 8 & 15 & 10 & 1 & 14 \\ 12 & 12 & 6 & 14 & 1 \end{bmatrix} \text{ and matrix } \mathbf{N}^{(2)} \begin{bmatrix} 1.1 & 8.1 & 7.1 & 8.1 & 12.1 \\ 8.1 & 1.1 & 7.1 & 15.1 & 12.1 \\ 7.1 & 7.1 & 1.1 & 10.1 & 6.1 \\ 8.1 & 15.1 & 10.1 & 1.1 & 14.1 \\ 12.1 & 12.1 & 6.1 & 14.1 & 1.1 \end{bmatrix}$$

We assume the regulator set the same weight to all other nodes (3), i.e. the node 4 will be tree time more attractive for the followers of the regulator.

If we do not consider lost demand, it is possible to quantify the elements of matrix **A** for player 1 as follows:

$$\mathbf{A} = \begin{bmatrix} 3 & 2.4 & 1.2 & 2.4 & 1.8 \\ 1.8 & 3 & 0.6 & 2.4 & 1.2 \\ 3.8 & 4.4 & 5 & 4.4 & 4.4 \\ 0.6 & 1.2 & 0.6 & 3 & 1.2 \\ 1.2 & 1.8 & 0.6 & 1.8 & 3 \end{bmatrix}$$

Since matrix **A** has a saddle point, there is only one solution to the game. The strategy of player 1 is represented by the vector: $\mathbf{x}^{(0)} = (0; 0; 1; 0; 0)$, i.e. the player 1 should invest in node 3 as the regulators prefers. The strategy of the player 2 is represented by the vector $\boldsymbol{y}^{(0)} = (1; 0; 0; 0; 0)$. As we can see, the player 2 in reaction of the action of player 1 should invest in node 1.

Obviously, the constant of this game is $c = 5$, the value of the game is 3.8 for player 1 (3.8 serviced nodes) and the value of the game for player 2 is $\mathrm{c} - 3.8 = 5 - 3.8 = 1.2$ (3 serviced nodes).

As a result, we can conclude that if the player 1 invests all capital in node 3 following the strategy of the regulator, he could take 3.8 node of five.

Let´s set maximum purchase price differences of the product $\mathrm{T_{max}} = 1$. The calculations are based on (4). In such case, if the total price (transport cost and product price) is lower or equals to 2 MU, and the player 1 is located in preferred node, the player 1 takes all demand from the preferred node plus $\lambda \in (0,1)$ of other nodes. In our case, the awareness level is $\lambda = 0.4$. It means that 40 % of all nodes follow the regulator and purchase in preferred node. On the other hand, if the player 1 is located in any node other than the preferred one and the total price differences is lower or equals to 1 MU, the player 1 only takes $1 - \lambda$ of the demand of such node.

The regulator prefers the node 3 as in previous example and we assume that $\lambda \in (0,1)$ is 0.40.

Preserving the previous prepositions matrix **A** is as follows:

$$\mathbf{A} = \begin{bmatrix} 3 & 1.2 & 0 & 0 & 0 \\ 1.2 & 3 & 0 & 0 & 0 \\ 3 & 4 & 5 & 4 & 4 \\ 0 & 0.6 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$$

Since matrix **A** does not have a saddle point, we search for a solution to the game in mixed strategies. The mixed equilibrium strategy of player 1 is represented by the vector: $\mathbf{x}^{(0)} = (0.25; 0; 0.75; 0; 0)$, i.e. the player 1 should invest in node 3, (around 75% in this location), the other good investment could be in node 1 with 25%. The reaction of the player 2 is represented by the vector $\mathrm{y}^{(0)} = (1; 0; 0; 0; 0)$ as we can see; the player 2 in reaction of the action of player 1 should invest in node 1. The value of the game is 3 for player 1 (3 serviced nodes) and value of the game for player 2 will be $c - 3 = 5 - 3 = 2$. (2 serviced nodes).

As we can see in this illustrative example, the participation of the regulator changes the strategy of player 1 and has an impact on the value of the game. In examples above we assumed unit demand. In case the calculations are extended to include real demand represented by the number of people living in a specific area, the impact of the regulator´s preferred node of the network will be more significant. By adding a parameter of different demand in various nodes, the game would change to a non-constant sum game. Solving such types of games can be found in (Čičková & Zagiba, 2018)

The games are formulated as a linear programming problem solved with GAMS (solver CONOPT 3 24.9.2 r64480).

5. CONCLUSION

Game theory can be used to solve them specific problem of spatial competition. Our paper is focused on the case where a regulatory entity is involved in the game. The problem is formulated for duopoly (on the supply side). The issues are analyzed in the transport network with individual buyers located in the individual nodes of such network. The sellers decide on their position while trying to respect the behavior of buyers who minimize both the costs associated with the transport price and the transport costs. The buyers react to the strategy of the regulatory entity; in this case also the sellers are bound to follow that. It is obvious that if the level of follower of the regulatory entity is high enough, the sellers will set up their branches in the same nodes as preferred by the regulator. If that percentage decreases or is almost zero or there is not a regulator, it is the situation formulated in (Sequeira Lopez & Čičková, 2018). The GAMS professional software, which ranks among the powerful optimization computing environments, was used to solve the games mentioned above.

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