A MIQP MODEL FOR SOLVING THE VEHICLE ROUTING PROBLEM WITH DRONES

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Abstract: *The presence of drones in everyday life expands on a daily basis. In the last couple of years, the usage of aerial drones (UAV-Unmanned Aerial Vehicles) in last-mile parcel delivery attracts more and more attention. Some companies (mainly in the USA and Australia) have already tested and applied the usage of drones in the parcel delivery. There are many papers describing a two-phase approach for routing the drone–ground vehicle tandem. Most of the previous work in this domain propose different heuristics, metaheuristics and optimization approaches for the transformation of a given truck route to a truck-drone route. Considering the simultaneous approach for solving the routing problem with drones, the literature is very scarce. The purpose of this paper is to present a novel MIQP (Mixed Integer Quadratic Programming) model of a simultaneous approach to solving the VRPDTW (Vehicle Routing Problem with Drones and Time Windows).*

Keywords*: MIQP, VRP, Unmanned Aerial Vehicles, Last-mile parcel delivery.*

1. INTRODUCTION

The usage of drones finds its place in many different spheres of everyday life: military purposes, surveillance, sports and recreation and of course logistics. The two main domains of drone application in logistics are distribution and warehousing. In warehousing, the drones are used for scanning purposes, while in the distribution the drones find their application in small parcel delivery. The application of drones in parcel delivery results in different kind of problems that must be solved: regulatory and safety problems, technical problems and the most interesting group - the tactical and operating problems (in this case, the routing problem). From 2015 and up to today, many articles and papers described various approaches for solving routing problems with drones. The purpose of this paper is to present a novel Mixed Integer Quadratic Programming (MIQP) model for solving the Vehicle Routing Problem with Drones and Time Windows (VRPDTW) with simultaneous optimization of both vehicle and drone operations. In the following sections of this paper we present a literature review on the topic of VRP with

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drones, problem formulation, the MIQP model, test instances with computational results and the concluding remarks.

2. LITERATURE REVIEW

The first papers proposing a solution to routing problem with drones appeared in 2015. Murray and Chu (2015) described a new subtype of the classical TSP (Traveling Salesman Problem) and named it FSTSP (Flying Sidekick Traveling Salesman Problem). In their paper, the authors propose a MILP model for the transformation of a given truck route to a truck-drone route. Later, in the same year, Agatz et al. (2015) proposed another MILP model for solving the so-called TSPD (Traveling Salesman Problem with Drones). These two papers were the pioneers of research in the area of solving routing problems with drones, so all other papers in the following years were related to the aforementioned research. In literature, there are many papers proposing different heuristics and metaheuristics for improving the approaches described in two aforementioned papers. Ponza (2016) showed that the application of SA (Simulated Annealing) can lead to much greater savings of the traveling time, compared to the original FSTSP. Ponza also proved that different technical characteristics of drones affect the setup of models for solving the routing problem with drones. Marinelli et al. (2017) describe an interesting approach for improving the initial TSPD solution, called “en-route” approach. All of the previous work assumed that the drone can leave/join the truck only at the customer nodes that are visited by the truck. The idea of “en-route” approach is to examine the effects of the concept where the drones could be launched from any point along an arc that the truck is traversing. Also, drones could join the truck again at any point along an arc that the truck is traversing. The authors proposed a heuristic for the “en-route” approach and proved that this approach could lead to significant savings in traveling time, due to the reduction of the total waiting time. The possible problem with this approach comes from the fact that sometimes the truck is not allowed to stop along the arcs. Savurhan and Karakaya (2015) developed a genetic algorithm for solving the TSPD. Luo et al. (2017) described a two-echelon approach for the TSPD, and they named it 2E-GU-RP (2 Echelon – Ground Vehicle and Unmanned Aerial Vehicle – Routing Problem). In the 2E-GU-RP the truck serves as a mobile warehouse, while all the nodes are served by the drones. When the truck stops at a certain location, all the nodes (that are in the flight range) are being served by drones. This approach could have a practical application in scenarios in which many of the customer nodes could not be visited by the ground vehicle. Ferrandez et al. (2016) compared the effects of truck-only, truck-drone tandem and drone-only (2E-GU-RP) parcel delivery.

Although Murray and Chu (2015) and Agatz et al. (2015) propose a one-phase approach for solving a one-vehicle routing problem with drones, to the best of our knowledge there is no previous work considering the one-phase approach for solving a routing problem with multiple vehicles and drones. Accordingly, the goal of this paper is to present a novel MIQP model of a systematic approach for solving the VRPDTW. The following chapters will describe and analyze the application of the proposed MIQP model for solving the VRPDTW.

3. PROBLEM FORMULATION

The goal of VRPDTW is finding the joint vehicles and drones routing solution with minimal cost for delivery from a single depot to a given set of customer locations with defined time windows, while some of the customer locations could be visited and served by a drone (while all locations can be visited by vehicles). In this paper, a heterogeneous fleet of vehicles is considered, which means that some of the vehicles contain a drone and some of them do not. The drone can depart/join the ground vehicle only at a customer node, while every drone is bound to its ground vehicle (it is not allowed for the ground vehicles to exchange drones). The proposed approach is cost-based, so the goal is to construct the least expensive solution, while the costs consist of traveling costs and labour costs.

4. MATHEMATICAL FORMULATION OF THE MIQP

The MIQP formulation in this paper is a derivate of the three-index MILP formulation by Fisher and Jaikumar (1978) given for solving the classical VRP. Let *V* be the set of all available ground vehicles, while V_k is the set of ordinary vehicles and V_d is the set of vehicles with drones ($V_k \cup V_d = V$). Let *I* be the set of all customer nodes, while *K* is the set of nodes that could potentially be visited by a drone $(K \subset I)$. The binary variable X_{ij}^{v} equals 1 if the vehicle *v* is traversing the arc *i-j*, and equals 0 if otherwise. The second binary variable X_{ikj}^{v} equals 1 if the vehicle *v* traverses the arc *i-j*, while the vehicle's drone traverses along the *i-k-j* section, otherwise, it equals 0. For each *i,j* pair there is a continuous variable C_{ij} that represents the distance between *i* and *j,* while C_{ikj} represents the distance *i-k-j*. Let T_v be a continuous variable that represents the route completion time for vehicle *v*. Analogously to the C_{ij} variables, t_{ij} variables represent the traveling time between each *i,j* pair of nodes, while t_i is an auxiliary variable that represents the moment when the node *i* is visited by a vehicle. The variable t_{ikj} represents the time needed for the drone to traverse the *i-k-j* section. Also, let Y_i^v be 1 if the node *i* is visited by vehicle *v,* 0 if otherwise. The objective function (1) aims to minimize the overall cost, while it distinguishes between the labour costs and traveling costs. Labour cost is calculated as a product of labour cost per minute – α, and the sum of all route completion times (for every vehicle). The traveling costs vary according to the vehicle type: β represents the distance unit cost for classical vehicles, η represents the distance unit cost of drone carrying vehicles, while ω represents the distance unit cost of drones. All the aforementioned results in the following MIQP formulation:

$$\min \sum_{v\in V} \alpha \cdot T_v + \sum_{i\in I}\sum_{j\in I}\sum_{v\in V_k} \beta \cdot C_{ij} \cdot X_{ij}^{v} + \sum_{i\in I}\sum_{j\in I}\sum_{v\in V_d} \eta \cdot C_{ij} \cdot X_{ij}^{v} + \sum_{i\in I}\sum_{j\in I}\sum_{k\in K}\sum_{v\in V_d} \left(\eta \cdot C_{ij} + \omega \cdot C_{ikj}\right) X_{ikj}^{v} \tag{1}$$

such that:

$$\sum_{v=1}^{m} Y_i^{v} = \begin{Bmatrix} 1, & \forall i \in I \backslash 0 \\ m, & i=0 \end{Bmatrix} \tag{2}$$

$$\sum_{i\in I}\sum_{j\in I}\sum_{k\in K} t_{ikj}\cdot X_{ikj}^{v} \le T_{\max}^{d} \qquad i\ne j\ne k,\ \forall v\in m \qquad (3)$$

$$t_0 = 0 \qquad (4)$$

$$t_i + t_{i0} \le T_{\max}^{kam} \qquad \forall i\in I \qquad (5)$$

$$t_j \ge t_i + T_{opsluge} + t_{ij} - \left(1-\left(\sum_{v=1}^{m} X_{ij}^{v} + \sum_{v\in Vd}\sum_{k\in K} X_{ikj}^{v}\right)\right)\cdot T_{\max}^{kam} \qquad \forall i,j\in I,\ i\ne j \qquad (6)$$

$$t_j \le t_i + T_{opsluge} + t_{ij} + \left(1-\left(\sum_{v=1}^{m} X_{ij}^{v} + \sum_{v\in Vd}\sum_{k\in K} X_{ikj}^{v}\right)\right)\cdot T_{\max}^{kam} \qquad \forall i,j\in I,\ i\ne j \qquad (7)$$

$$t_{ij} \ge \sum_{v\in V} X_{ij}^{v}\cdot\frac{C_{ij}}{V_{kam}} \qquad \forall i,j\in I,\ i\ne j \qquad (8)$$

$$t_{ij} \ge \sum_{v\in Vd} X_{ikj}^{v}\cdot\frac{C_{ij}}{V_{kam}} \qquad \forall i,j\in I,\ \forall k\in K,\ i\ne j\ne k \qquad (9)$$

$$t_{ij} \ge \sum_{v\in Vd} X_{ikj}^{v}\cdot\frac{C_{ikj}}{V_{drona}} \qquad \forall i,j\in I,\ \forall k\in K,\ i\ne j\ne k \qquad (10)$$

$$a_i \le t_i \le b_i \qquad \forall i\in I \qquad (11)$$

$$Y_i^{v} = \begin{cases} \sum_{j\in I} X_{ij}^{v} + \sum_{j\in I}\sum_{k\in K} X_{ikj}^{v}, v\in Vd \\ \sum_{j\in I} X_{ij}^{v}, v\notin Vd \end{cases} \qquad \forall i\in I,\ \forall \mathrm{v}\in \mathrm{m},\ \mathrm{i}\ne \mathrm{j}\ne \mathrm{k} \qquad (12)$$

$$Y_i^{v} = \begin{cases} \sum_{j\in I} X_{ji}^{v} + \sum_{j\in I}\sum_{k\in K} X_{jki}^{v}, v\in Vd \\ \sum_{j\in I} X_{ji}^{v}, v\notin Vd \end{cases} \qquad \forall i\in I,\ \forall \mathrm{v}\in \mathrm{m},\ \mathrm{i}\ne \mathrm{j}\ne \mathrm{k} \qquad (13)$$

$$T_v \ge \begin{cases} (t_{i0}+t_i)\cdot X_{i0}^{v},\ \forall \mathrm{v}\in \mathrm{V}_k \\ (t_{i0}+t_i)\cdot (X_{i0}^{v} + \sum_{k\in K} X_{ik0}^{v}),\ \forall \mathrm{v}\in \mathrm{V}_d \end{cases} \qquad \forall i\in I\setminus 0,\ \mathrm{i}\ne \mathrm{k} \qquad (14)$$

$$W_d^{ikj} \ge X_{ikj}\cdot\left(\frac{C_{ij}}{V_{kamiona}} - \frac{C_{ikj}}{V_{drona}}\right) \qquad \forall i,j\in I,\ \forall k\in K,\ i\ne j\ne k \qquad (15)$$

$$W_d^{ikj} \ge 0 \qquad \forall i,j\in I,\ \forall k\in K,\ i\ne j\ne k \qquad (16)$$

$$W_v^{ikj} \ge X_{ikj}\cdot\left(\frac{C_{ikj}}{V_{drona}} - \frac{C_{ij}}{V_{kamiona}}\right) \qquad \forall i,j\in I,\ \forall k\in K,\ i\ne j\ne k \qquad (17)$$

$$W_v^{ikj} \ge 0 \qquad \forall i,j\in I,\ \forall k\in K,\ i\ne j\ne k \qquad (18)$$

Constraint (2) ensures that every node must be visited by a vehicle exactly once, while the depot must be visited by all vehicles *V*. Constraint (3) ensures that the drone flight would not exceed the maximum drone flying time $T_d^{\max}$. Constraint (4) sets the depot departure time, and sets the beginning moment of every vehicle route (0 s), while the constraint (5) ensures that every vehicle has enough time to return to the depot in the limits of the allowed maximum route duration time ($T_{kam}^{\max}$). Constraints (6) and (7) define the visiting moment of every node, and together with the constraint (5) eliminate all sub-tours. The variable $T_{opsluge}$ represents the time needed for serving the customer upon the arrival to the customers' location. Constraints (8), (9) and (10) set the values for the time needed for traveling between two nodes, whether the *i-j* distance is travelled only by a ground vehicle, or the distance *i-j* is traversed by a ground vehicle, while the drone traverses the *i-k-j* distance. In the case of drone assistance, the time t_{ikj} will be equal to the greater value of the time needed for the truck to traverse the distance *i-j*, and the time that it takes for the drone to traverse *i-k-j*. Constraint (11) ensures that no time-window is violated, while (12) and (13) ensure that every node has exactly one inbound and one outbound arc. Equation (14) sets the corresponding values to the variables that represent the time needed for a vehicle to complete its tour. Constraints (15), (16), (17) and (18) calculate the drone and truck waiting times for every *i-k-j* arc that is served by a truck-drone tandem.

5. INSTANCE GENERATION AND MIQP MODEL SETUP

This section will explain the instances, input parameters' values and MIQP model setup. There are several types of instances and they are shown in Table 1. Every instance consists of 10 nodes and a depot. There are two variants of depot location: in the first variant the depot is located at [0,0], and in the second variant the depot is located in the center of the defined area. There are two instance area sizes: 30 km x30 km and 60 km x60 km. Also, two variants of node structures are considered: the first variant consists of nodes where half of them could be visited by the drone, and in the second variant, all nodes could be visited by the drone. All nodes are randomly distributed in the defined area, with randomly generated time-windows within the boundary time-windows range. There are three considered types of time-windows range: 60 min, 120 min and 480 min. Regarding the MIQP model input parameters, the labour costs value is set to 0.05 €/min, while the distance unit cost values for classical vehicles, drone carrying vehicles and drones are set to 1.0, 1.5 and 0.2 €/min respectively. $T_{opsluge}$ is set to be 10 minutes. The speed of the ground vehicles is set to be 40 km/h, while the speed of drones is set to 60 km/h. The maximum allowed route duration time per vehicle is set to be 8 hours, while the maximum allowed CPU for solving an instance is set to be 30 min.

6. RESULTS ANALYSIS

In this chapter, the result analysis of the MIQP model application will be presented. The following output parameters and results will be analyzed: the average objective function value, the average CPU, the average distance travelled, the average vehicle traveling time, the average vehicle waiting time and the average number of used vehicles. All the aforementioned parameters will be analyzed and compared from the aspect of different vehicle types and the maximum drone flight range. The model solved all instance with optimality within given CPU time restriction, except for instance types 4 and 16 when the maximum drone flight time is set up to be 60 minutes.

Table 1. Instance types

Instance type	Depot location	Area dimensions (km x km)	Percentage of drone nodes	Time windows range (min)
Type 1	[0,0]	30x30	50%	120
Type 2	[0,0]	30x30	100%	120
Type 3	[0,0]	30x30	50%	480
Type 4	[0,0]	30x30	100%	480
Type 5	[0,0]	30x30	50%	60
Type 6	[0,0]	30x30	100%	60
Type 7	[0,0]	60x60	50%	120
Type 8	[0,0]	60x60	100%	120
Type 9	[0,0]	60x60	50%	480
Type 10	[0,0]	60x60	100%	480
Type 11	[0,0]	60x60	50%	60
Type 12	[0,0]	60x60	100%	60
Type 13	center	30x30	50%	120
Type 14	center	30x30	100%	120
Type 15	center	30x30	50%	480
Type 16	center	30x30	100%	480
Type 17	center	30x30	50%	60
Type 18	center	30x30	100%	60
Type 19	center	60x60	50%	120
Type 20	center	60x60	100%	120
Type 21	center	60x60	50%	480
Type 22	center	60x60	100%	480
Type 23	center	60x60	50%	60
Type 24	center	60x60	100%	60

From Table 2 it can be noticed that the average objective function had a lower value in the model variant where T_d^{max} =60 min. However, a better objective function value lead to greater CPU times. The conclusion that can be made is that the cost of the truck-drone parcel distribution decreases with the improvement of the technical characteristics of the drone (in this case, the drone flight range). The average objective function value for all instances and both model setups is 217.5 €, while the average CPU is 100 s. Comparing the two T_d^{max} input values, the MIQP model gave better cost savings when all the nodes were available for the drone to visit with T_d^{max} =60 min. The average cost savings of T_d^{max} being 60 instead of 30 min is 7.23%, while the average increase of CPU needed for solving the instances is 277 s. The only instance type where the model with T_d^{max} = 30 min outperforms the 60 min version is type 4. The reason this occurred is that the model could not solve some instances to optimality within the given CPU time restriction when T_d^{max} was set to be 60 min.

The second parameter that will be analyzed is the average distance travelled by a vehicle type. The average values per different instance types are shown in Figures 1 and 2. The average distance travelled for the case T_d^{max} = 30 min is 151 km for the classic truck, 20 km for the drone carrying truck and 26 km for the drone. The average distance travelled for the case T_d^{max} = 60 min is 92 km for the classic truck, 45 km for the drone carrying truck and 81 km for the drone. The average reduction of classic truck route length is 39%, and the average increase of drone carrying truck and drone routes are 122% and 212% respectively when changing the value of T_d^{max} from 30 to 60 min.

Table 2. Average objective function value and average CPU by instance for T_d^{max} of 30 and 60 min

Instance type	T_d^{max} = 30 min Avg. objective function value (€)	T_d^{max} = 30 min Avg. CPU time (s)	T_d^{max} = 60 min Avg. objective function value (€)	T_d^{max} = 60 min Average CPU time (s)	Difference of 60 min from 30 min T_d^{max} solutions Obj. func. (%)	Difference of 60 min from 30 min T_d^{max} solutions CPU time (%)
Type 1	167.52	12.26	165.64	37.67	-1.12	207.26
Type 2	156.20	28.66	149.42	312.27	-4.34	989.57
Type 3	119.44	183.53	119.44	388.15	0.00	111.49
Type 4	119.44	340.74	120.39*	1262.60	0.80	270.55
Type 5	207.28	2.03	199.65	5.69	-3.68	180.30
Type 6	170.40	3.91	158.95	24.33	-6.72	522.25
Type 7	375.29	4.51	361.15	8.81	-3.77	95.34
Type 8	371.52	5.77	311.15	20.01	-16.25	246.79
Type 9	233.88	34.12	233.88	113.11	0.00	231.51
Type 10	233.88	60.87	233.88	259.66	0.00	326.58
Type 11	418.05	0.88	404.90	1.82	-3.15	106.82
Type 12	411.38	1.30	355.98	4.16	-13.47	220.00
Type 13	148.78	12.06	146.18	35.86	-1.75	197.35
Type 14	131.25	67.15	117.65	131.67	-10.36	96.08
Type 15	107.84	34.42	106.27	124.98	-1.46	263.10
Type 16	105.20	148.20	98.96*	848.24	-5.93	472.36
Type 17	161.90	2.66	157.91	9.18	-2.46	245.11
Type 18	135.72	6.52	120.02	25.69	-11.57	294.02
Type 19	291.90	2.28	271.33	6.80	-7.05	198.25
Type 20	288.60	4.57	237.75	19.58	-17.62	328.45
Type 21	215.61	10.22	210.92	44.03	-2.18	330.82
Type 22	215.61	40.00	206.25	118.99	-4.34	197.48
Type 23	318.29	0.88	293.72	1.46	-7.72	65.91
Type 24	313.15	1.43	244.75	2.99	-21.84	109.09
Total avg.	225.76	42.04	209.42	158.66	-7.23	277.39

**-some MIQP solutions were not solved to optimality in available computational time*

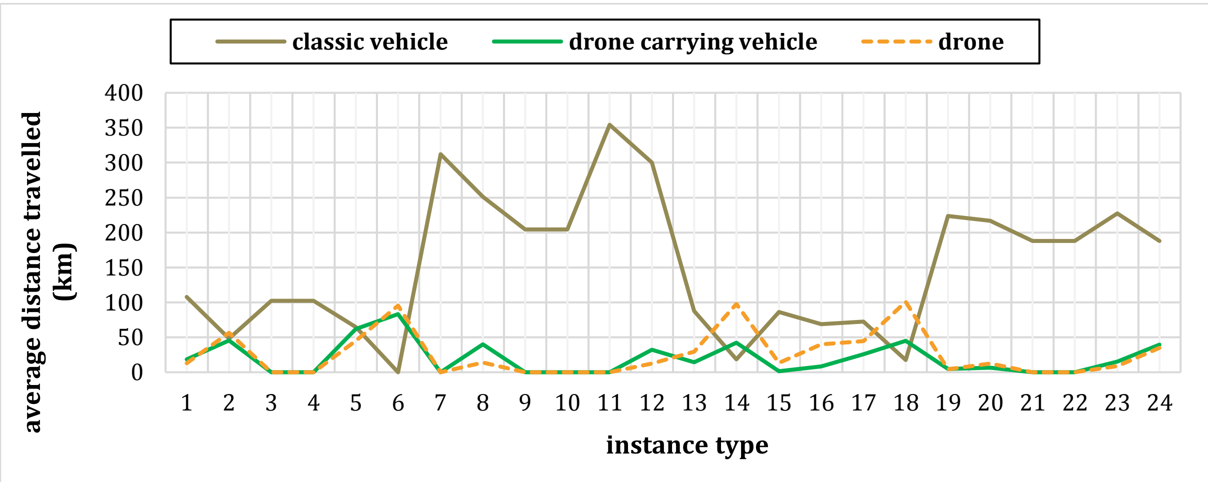

Figure 1. Average distance travelled by vehicle type, T_d^{max} = 30 min

The vehicle route lengths were drastically reduced with the increase of the maximum allowed drone flight time. As expected, the intervention of drone is greater in the instances where the drone is allowed to visit all of the nodes. In instance types 3, 4, 9 and 10, for both T_d^{max} values, all the nodes were visited by the classic truck, and no drone was used for parcel delivery in these instance types. The classic vehicle was never used in instance type 6. So, the conclusion is that the traveling distance and the usage of different vehicle types is strongly dependent on the instance structure, as well as the characteristics of the drone.

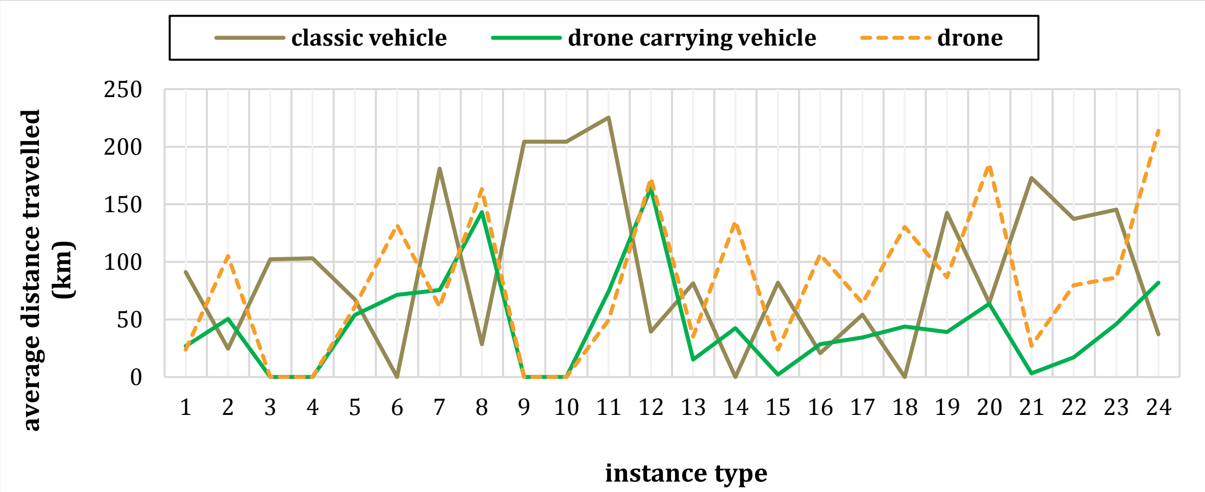

Figure 2. Average distance travelled by vehicle type, T_d^{max} = 60 min

Figures 3 and 4 show that the average time travelled per vehicle type varies for different instance types. All the results of traveling time are related to the results of the distance travelled by each vehicle type. Considering the waiting times, the average waiting time of the truck was 11.15 min (with T_d^{max} = 30 min) and 38.72 min (with T_d^{max} =60 min. The drone waiting time had always a lesser value than the truck waiting time, except on the instance type 12 and T_d^{max} = 60 min.

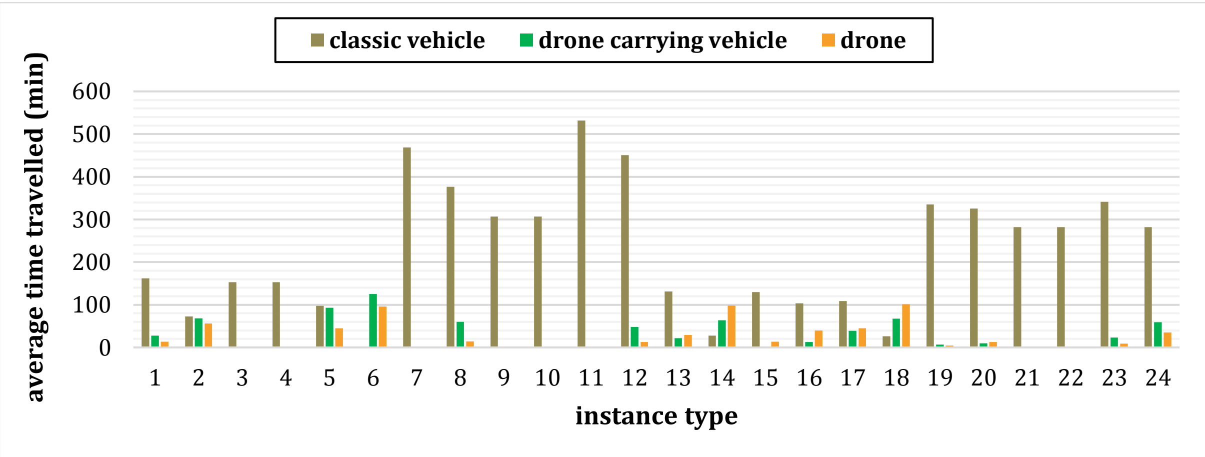

Figure 3. Average time travelled by vehicle type, T_d^{max} = 30 min

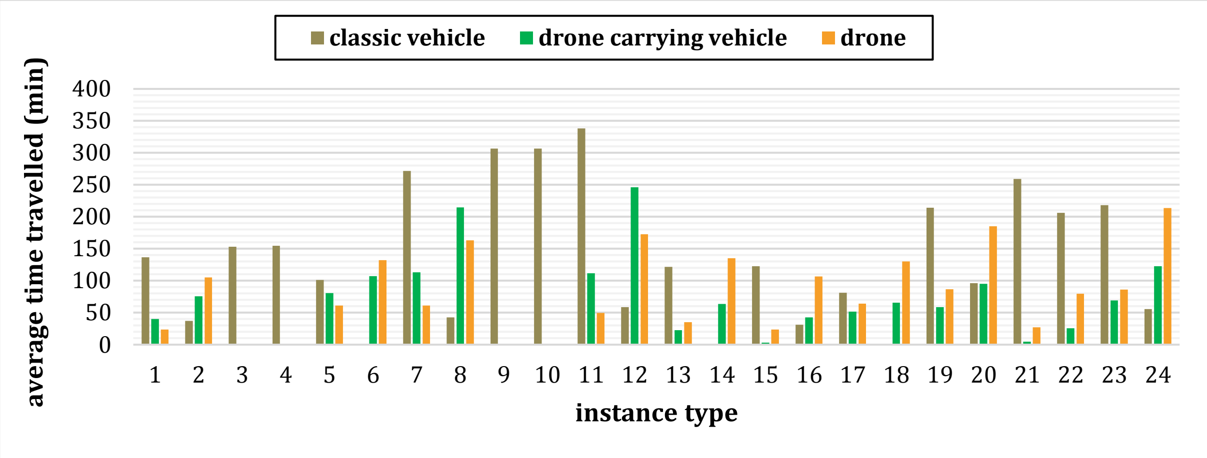

Figure 4. Average time travelled by vehicle type, $T_d^{\max}$ = 60 min

Considering the average number of used vehicles, the increase of the allowed drone flight time results in a more frequent usage of drone-carrying trucks in parcel delivery. The average number of used vehicles (both classic and ones carrying drone) when $T_d^{\max}$ = 30 min is 1.5, while the average number of used vehicles when $T_d^{\max}$ = 60 min was 1.4. Based on this result, we can conclude that the usage of drones with better technical characteristics could lead to a reduction in the number of ground vehicles needed for distribution.

Different route structures for the same instance, with a different number of drone nodes, and different $T_d^{\max}$ are shown in Figure 5. It is obvious that the route structures are strongly dependent on the allowed drone flight time and the number of locations that could be visited by the drone.

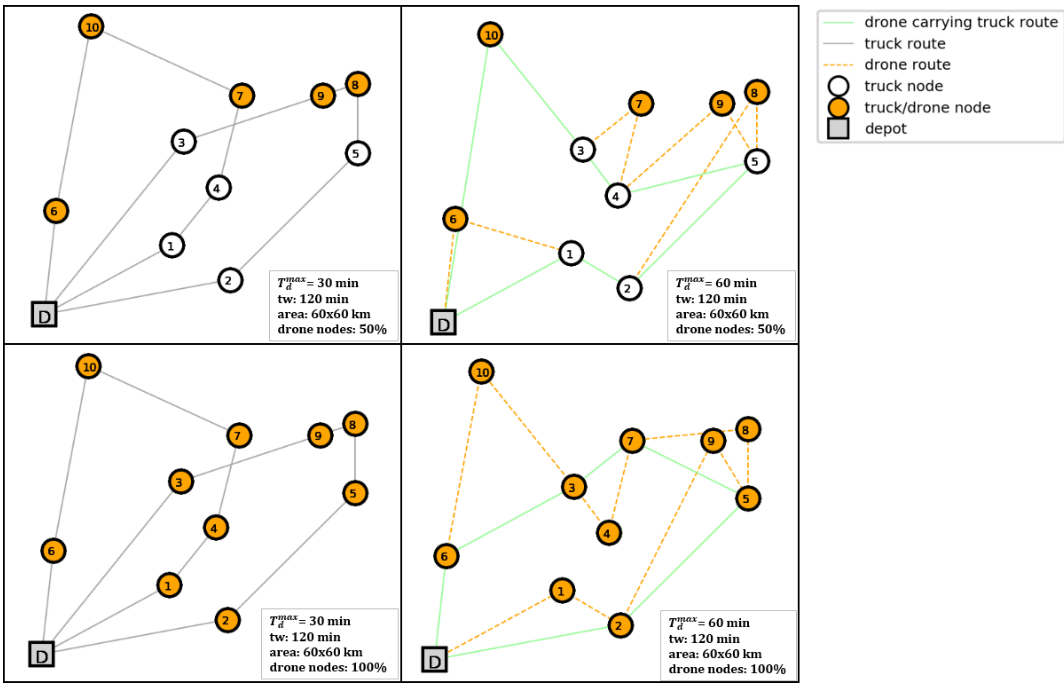

Figure 5. Different route structures per some instance types

7. CONCLUSION

In this paper, a novel MIQP model for solving the VRPDTW in a systematic manner is presented. The model was formulated to solve various types of costs in a tandem vehicle-drone parcel delivery, with the goal of minimizing the distribution cost related to travelled distance and working time. The model was used to solve various types of instances. The application results have shown that a significant cost saving could be achieved with the intervention of drones in parcel delivery. Further research could go in several directions. The first direction could be exploring the effects of the approach where the ground vehicles could exchange drones so that no drone is bound to a particular truck. The other research directions could focus on how the improvement of drone technical characteristics could affect the distribution and logistics in general. Also, a development of a heuristic approach to solve larger scale instances that are closer to real-life dimensions is one of the interesting research possibilities.

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